Christ the King Diocesian HIgh School
Incoming Precalculus
Summer Math Packet

This packet will help you review basic algebra concepts.

- Please show all your work. No work No Credit!!!
  (if you need more room use loose leaf paper to do your work and staple it to the corresponding worksheet)
- You will be expected to do a worksheet every week.
- I have provided notes with worked examples to help you.
- Please join my Incoming Precalculus Summer Math Google Classroom by entering the following code: **5pqjamu,** as I will include helpful videos there to help you complete these assignments.
- Do not wait to do all of the worksheets at one time.
- This packet will be due Wednesday August 16, 2023

Proposed schedule

Worksheet		Date: Week of
Week 1	3-4 problems/day	June 5
Week 2	2 problems/day	June 12
Week 3	1-2 problems/day	June 19
Week 4	2 problems/day	June 26
Week 5	3 problems/day	July 3
Week 6	2 problems/day	July 10
Week 7	3-4 problems/day	July 17
Week 8	3 problems/day	July 24

Incoming Precalculus Summer Math Name________________________ ID: 1

Week 1 - Exponent/Polynomial Operations (3-4 Problems/day)

Simplify. Your answer should contain only positive exponents.

1) $2b^3 \cdot 2b^{-4}$

2) $3x^3 \cdot 3x^4$

3) $4x^2 \cdot x^{-4}y^2$

4) $3nm^3 \cdot -4m^{-4}n^3$

5) $\left(-2x^2 \cdot 2x^4\right)^2$

6) $\left(-3x^3\right)^4$

7) $\left(2k^2\right)^4$

8) $\left(4n^2\right)^3$

9) $\dfrac{2x^4y^4}{4x^{-1}y^2}$

10) $\dfrac{3x^4y^{-4}}{3yx^{-2}}$

Simplify each expression.

11) $\left(-2n + 4n^4 + 6n^2\right) - \left(-5n^4 + 2n + 6n^2\right)$

12) $\left(-7x - 8x^2 + 4x^4\right) + \left(2x + 4x^4 + 6\right)$

13) $\left(-\dfrac{1}{4}k^2 + \dfrac{3}{2}k\right) - \left(-\dfrac{10}{3}k^2 - \dfrac{13}{4}k\right)$

Find each product.

14) $(-7b + 3)(8b - 1)$

15) $(2x + 4)(8x - 5)$

16) $(-4x - 2)(5x + 8)$

17) $(x + 5)(8x + 3)$

18) $(-5b + 2)(7b^2 - 8b - 1)$

19) $(-4k + 5)(2k^2 - 4k + 6)$

Name each polynomial by degree and number of terms.

20) $9m^3$

21) -2

22) $10n$

23) $3x^4$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
MONOMIALS	• A **monomial** is a <u>number</u>, a <u>variable</u> or a <u>product of a number and variable(s)</u> • **Examples:** $5x,\ 3x^2,\ 9x^2y^7,\ -\frac{5}{2}xy^4,\ -\frac{6}{11}$ • Monomials with the **same variables and exponents** are <u>like terms</u>!
ADDING & SUBTRACTING MONOMIALS	• To **add or subtract monomials**, <u>combine like terms</u>! • **DO NOT CHANGE** the variables and exponents!

Directions: Add or subtract the following monomials.

1. $19x-24x$ $-5x$	**2.** $-19k^2+2k^2$ $-17k^2$
3. $-12r^2s^3t+4r^2s^3t$ $-8r^2s^3t$	**4.** $27ab^2-ab^2$ $26ab^2$
NOT like terms! → **5.** $7ab+4ac$ $7ab+4ac$	**6.** $-7cd+6cd$ $-cd$
7. $\frac{2}{3}x^2y-\frac{1}{4}x^2y$ $\frac{5}{12}x^2y$	**8.** $-15m^2n^4-15m^2n^4$ $-30m^2n^4$
9. $2y+14z-11y-10z$ $-9y+4z$	**10.** $16s+8r-17s$ $8r-s$
11. $11v^2+7v+2v^2-32v$ $13v^2-25v$	**12.** $9a^2b+5ab^2-a^2b$ $8a^2b+5ab^2$
13. Find the sum of $14z^2$ and $-9z^2$. $5z^2$	**14.** Find the difference of $13a^2b$ and $-5a^2b$. $13a^2b-(-5a^2b)=18a^2b$
15. Subtract $3k$ from $-17k$. $-17k-3k=-20k$	**16.** From $-3xy^2$, subtract $-10xy^2$. $-3xy^2-(-10xy^2)=7xy^2$

MULTIPLYING MONOMIALS	To multiply monomials, use the **PRODUCT RULE:**		$x^a \cdot x^b = x^{a+b}$
	Directions: Find each product.		
	17. $x^4 \cdot x^2$ x^6	**18.** $a^4 \cdot a$ a^5	**19.** $w^5 \cdot w^9$ w^{14}
	➢ **Examples with Coefficients:** • MULTIPLY the coefficients. • SIMPLIFY the variables with the product rule.		
	20. $(3x^3)\cdot(7x^2)$ $21x^5$	**21.** $(-8m^7)(2m^3)$ $-16m^{10}$	**22.** $9x \cdot 3x^7$ $27x^8$
	23. $(-6ab^2)(4a^3b^6)$ $-24a^4b^8$	**24.** $-2p^7q^5r \cdot (-9p^4q^2r^3)$ $18p^{11}q^7r^4$	**25.** $(6x^5y^8)(-7x^2y^4)$ $-42x^7y^{12}$
	26. $5y \cdot -3y^4 \cdot 4y^7$ $-60y^{12}$	**27.** $(mn)(8m^2n)(-mn^9)$ $-8m^4n^{11}$	**28.** $-4c^3d \cdot cd \cdot 7c^4d^2$ $-28c^8d^4$
	29. $9p \cdot 5p^2q \cdot q^4$ $45p^3q^5$	**30.** $(4x^5y)(12x^3y^8)$ $48x^8y^9$	**31.** $-6(gh^9)(-2g^3h)$ $12g^4h^{10}$
	32. $(-24x^2y)\cdot\left(\frac{1}{3}xy\right)$ $-8x^3y^2$	**33.** $16r \cdot \left(\frac{5}{4}q^2r^5\right)$ $20r^6q^2$	**34.** $\frac{1}{3}(2xy^3)(-15x^7y)$ $-10x^8y^4$
MIXED PRACTICE	**35.** $(3x^5y)(2x^4y^3) + (5x^7y^2)(x^2y^2)$ $6x^9y^4 + 5x^9y^4$ $11x^9y^4$		**36.** $(-3a^4b)(4ab^6) + (a^3b^5)(-7a^2b^2)$ $-12a^5b^7 + (-7a^5b^7)$ $-19a^5b^7$
	37. $17m^6n^{11} - (5m^5n^2)(4mn^9)$ $17m^6n^{11} - 20m^6n^{11}$ $-3m^6n^{11}$		**38.** $(-8c^5d)(-4c^3d^4) - (9c^7d^2)(3cd^3)$ $32c^8d^5 - 27c^8d^5$ $5c^8d^5$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
Powers of Monomials	To raise a monomial to a power, use the **POWER RULE:** $(x^a)^b = x^{a \cdot b}$
Examples	**Directions:** Simplify.

1. $(x^2)^8$	2. $(m^9n^5)^4$	3. $(pq^5)^3$
x^{16}	$m^{36}n^{20}$	p^3q^{15}

➢ **Examples with Coefficients:**

- Raise the coefficient to the given power.
- SIMPLIFY the variables with the power rule.

4. $(5x^7)^2$	5. $(2x^2y)^5$	6. $(-8c^4d^9)^2$
$25x^{14}$	$32x^{10}y^5$	$64c^8d^{18}$
7. $(-5m^6n)^4$	8. $(-3x^2)^3$	9. $\left(\frac{1}{2}a^3b^4c^5\right)^3$
$625m^{24}n^4$	$-27x^6$	$\frac{1}{8}a^9b^{12}c^{15}$

Mixed Practice

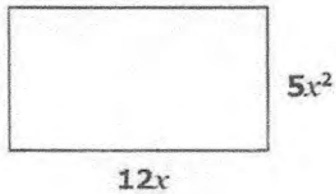
Directions: Simplify each expression completely.

10. $(x^3y^3)^3 \cdot xy^2$	11. $r^4 \cdot (-r^7s)^3$
$x^9y^9 \cdot xy^2$ $= \boxed{x^{10}y^{11}}$	$r^4 \cdot -r^{21}s^3$ $= \boxed{-r^{25}s^3}$
12. $(-ab^7)^2 \cdot (12a^3b)^2$	13. $(6a^2b)^3 \cdot \left(\frac{1}{3}abc\right)^2$
$a^2b^{14} \cdot 144a^6b^2$ $= \boxed{144a^8b^{16}}$	$216a^6b^3 \cdot \frac{1}{9}a^2b^2c^2$ $= \boxed{24a^8b^5c^2}$
14. $(2a^2)^3 + (a^4)(3a^2)$	15. $(3x^3y)^4 - (7x^5y)^2 \cdot x^2y^2$
$8a^6 + 3a^6$ $= \boxed{11a^6}$	$81x^{12}y^4 - 49x^{10}y^2 \cdot x^2y^2$ $81x^{12}y^4 - 49x^{12}y^4$ $= \boxed{32x^{12}y^4}$

Geometric Applications

Directions: Find the **perimeter** and **area** of each figure below.

16.



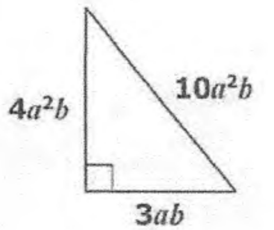
$P = 2(12x) + 2(5x^2)$

$= \boxed{24x + 10x^2}$

$A = 12x \cdot 5x^2$

$= \boxed{60x^3}$

17.



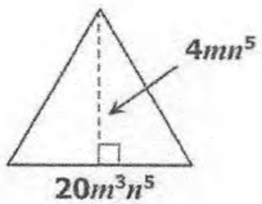
$P = 4a^2b + 10a^2b + 3ab$

$= \boxed{14a^2b + 3ab}$

$A = \frac{1}{2}(4a^2b)(3ab)$

$= \boxed{6a^3b^2}$

18.



$P = 3(20m^3n^5)$

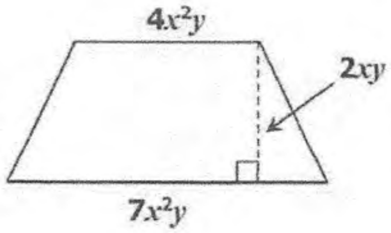
$= \boxed{60m^3n^5}$

$A = \frac{1}{2}(20m^3n^5)(4mn^5)$

$= \boxed{40m^4n^{10}}$

Directions: Find the **area** of each figure below.

19.

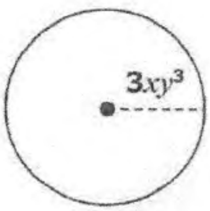


$A = \frac{1}{2}(4x^2y + 7x^2y)(2xy)$

$= \frac{1}{2}(11x^2y)(2xy)$

$= \boxed{11x^3y^2}$

20.



$A = \pi(3xy^3)^2$

$A = \boxed{9\pi x^2y^6}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
DIVIDING MONOMIALS	To divide monomials, use the **QUOTIENT RULE:** $\boxed{\frac{x^a}{x^b} = x^{a-b}}$

EXAMPLES

Directions: Find each quotient.

1. $\frac{x^5}{x^3}$ x^2	**2.** $\frac{k^{12}}{k^2}$ k^{10}	**3.** $\frac{m^3}{m^3}$ $m^0=1$
4. $\frac{a^6b^4}{a^2b^3}$ a^4b	**5.** $\frac{p^7q^{16}}{p^4q^{12}}$ p^3q^4	**6.** $\frac{x^{20}y^9z^2}{x^5y^9z}$ $x^{15}y^0z^1$ $=x^{15}z$

➢ **Examples with Coefficients:**

- DIVIDE the coefficients.
- SIMPLIFY the variables with the quotient rule.

7. $\frac{6x^4}{2x^3}$ $3x$	**8.** $\frac{14r^2s^2}{7rs}$ $2rs$	**9.** $\frac{-36c^2d^5}{4c^2d^3}$ $-9d^2$
10. $\frac{-15x^6y^5z}{-3x^5y^3}$ $5xy^2z$	**11.** $\frac{4n^5}{8n}$ $\frac{1}{2}n^4$	**12.** $\frac{36m^9n^5}{54m^3n^2}$ $\frac{2}{3}m^6n^3$

MIXED PRACTICE

Directions: Simplify each expression completely.

13. $\frac{(3x^5)^2}{27x^3}$ $\frac{9x^{10}}{27x^3}$ $=\boxed{\frac{1}{3}x^7}$	**14.** $\frac{(2a^2b^4)^3}{4a^3b^7}$ $\frac{8a^6b^{12}}{4a^3b^7}$ $=\boxed{2a^3b^5}$
15. $\frac{12w^9v^4}{(4wv)^2}$ $\frac{12w^9v^4}{16w^2v^2}$ $=\boxed{\frac{3}{4}w^7v^2}$	**16.** $\frac{(2cd^4)^4}{(2c^2d^3)^2}$ $\frac{16c^4d^{16}}{4c^4d^6}$ $=\boxed{4d^{10}}$

17. $\left(\frac{2m^4n^2}{4m^2n}\right)^3 = \frac{8m^{12}n^6}{64m^6n^3}$

$= \boxed{\frac{1}{8}m^6n^3}$

18. $\left(\frac{12k^5}{15k}\right)^2 = \frac{144k^{10}}{225k^2}$

$= \boxed{\frac{16}{25}k^8}$

19. $\left(\frac{4ab^2}{5ab}\right)^2 = \frac{16a^2b^4}{25a^2b^2}$

$= \boxed{\frac{16}{25}b^2}$

20. $\frac{(9x^5y^6)(4xy)}{6x^2y^4} = \frac{36x^6y^7}{6x^2y^4}$

$= \boxed{6x^4y^3}$

21. $\frac{(2pq)(3p^2q^4)}{-6pq^5} = \frac{6p^3q^5}{-6pq^5}$

$= \boxed{-p^2}$

22. $\frac{(10ab)^2(2a^4b^3)}{4a^5b} = \frac{200a^6b^5}{4a^5b}$

$= \boxed{50ab^4}$

23. $\frac{(2x^3)^2(3y^4)^3}{12x^4y^5} = \frac{108x^6y^{12}}{12x^4y^5}$

$= \boxed{9x^2y^7}$

24. $\frac{(3m^2)^2(-4n^5)^2}{8m^3n^4} = \frac{144m^4n^{10}}{8m^3n^4}$

$= \boxed{18mn^6}$

25. $\frac{(8cd^3)(-3c^4)}{6c^2d} - 9c^3d^2$

$\frac{-24c^5d^3}{6c^2d} - 9c^3d^2$

$-4c^3d^2 - 9c^3d^2 = \boxed{-13c^3d^2}$

26. $\frac{(8r^5s^2)(3r^3s^4)}{12rs^4} + 9r^7s^2$

$\frac{24r^8s^6}{12rs^4} + 9r^7s^2$

$2r^7s^2 + 9r^7s^2 = \boxed{11r^7s^2}$

27. $\frac{(2m^2)^3(3n^4)^3}{9m^4n^7} - 18m^2n^5$

$\frac{8m^6 \cdot 27n^{12}}{9m^4n^7} - 18m^2n^5$

$24m^2n^5 - 18m^2n^5 = \boxed{6m^2n^5}$

28. $\frac{(-6x^4y^6)^2}{(-3x^3y^5)^2} - 7x^2y^2$

$\frac{36x^8y^{12}}{9x^6y^{10}} - 7x^2y^2$

$4x^2y^2 - 7x^2y^2 = \boxed{-3x^2y^2}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
Negative Exponents	Negative exponents can be rewritten using positive exponents using the **NEGATIVE EXPONENT RULE:** $x^{-a} = \frac{1}{x^a}$
Examples	**Directions:** Rewrite each expression using positive exponents.

1. x^{-5} $\frac{1}{x^5}$	**2.** $3m^{-2}$ $\frac{3}{m^2}$	**3.** $-7a^{-4}b^3$ $\frac{-7b^3}{a^4}$

Directions: Simplify each expression. Make sure final answers contain only positive exponents. (**Hint**- use the rules, then move variables at the end!)

4. $w^7 \cdot w^{-9}$ w^{-2} $= \boxed{\frac{1}{w^2}}$	**5.** $4c^8d^{-3} \cdot 5c^{-5}d^{-1}$ $20c^3d^{-4} = \boxed{\frac{20c^3}{d^4}}$
6. $6x^{-8} \cdot -3x^{-3}$ $-18x^{-11} = \boxed{\frac{-18}{x^{11}}}$	**7.** $(a^{-5}b^8c^{-12})(a^7b^{-3}c^7)$ $a^2b^5c^{-5} = \boxed{\frac{a^2b^5}{c^5}}$
8. $(8p^5)^{-2}$ $\frac{1}{64}p^{-10} = \boxed{\frac{1}{64p^{10}}}$	**9.** $(5y^2)^{-3}$ $\frac{1}{125}y^{-6} = \boxed{\frac{1}{125y^6}}$
10. $(3rs^{-3})^{-4}$ $\frac{1}{81}r^{-4}s^{12} = \boxed{\frac{s^{12}}{81r^4}}$	**11.** $(6x^5y^{-4})^{-2}$ $\frac{1}{36}x^{-10}y^8 = \boxed{\frac{y^8}{36x^{10}}}$
12. $\frac{h^2}{h^5}$ $h^{-3} = \boxed{\frac{1}{h^3}}$	**13.** $\frac{c^{-2}d^{-1}}{c^7d^{-2}}$ $c^{-9}d^1 = \boxed{\frac{d}{c^9}}$
14. $\frac{14w^4}{7w^{-2}}$ $\boxed{2w^6}$	**15.** $\frac{-10m^2n}{2m^3n^{-5}}$ $-5m^{-1}n^6 = \boxed{\frac{-5n^6}{m}}$

	16. $\frac{36x^{-4}y^8}{12y^7}$ $3x^{-4}y$ $= \boxed{\frac{3y}{x^4}}$	**17.** $\frac{15a^2b^5c^8}{18a^2b^3c^9}$ $\frac{5}{6}b^2c^{-1}$ $= \boxed{\frac{5b^2}{6c}}$
	18. $\frac{-4pq^5r^3}{8p^2q^2r^{10}}$ $-\frac{1}{2}p^{-1}q^3r^{-7}$ $= \boxed{\frac{-q^3}{2pr^7}}$	**19.** $\frac{-9r^2s^6t^4}{54r^5s^2t^8}$ $-\frac{1}{6}r^{-3}s^4t^{-4}$ $= \boxed{\frac{-s^4}{6r^3t^4}}$
Mixed Practice	**20.** $(4x^3y^6)^{-2} + (2x^2y^4)^{-3}$ $\frac{1}{16}x^{-6}y^{-12} + \frac{1}{8}x^{-6}y^{-12}$ $\frac{3}{16}x^{-6}y^{-12} = \boxed{\frac{3}{16x^6y^{12}}}$	**21.** $(x^2y^3)^{-2}\cdot(x^5y^4)^{-3}$ $x^{-4}y^{-6}\cdot x^{-15}y^{-12}$ $x^{-19}y^{-18}$ $= \boxed{\frac{1}{x^{19}y^{18}}}$
	22. $\frac{(6a^3)(5a^9)}{-12a^{14}}$ $\frac{30a^{12}}{-12a^{14}}$ $-\frac{5}{2}a^{-2} = \boxed{\frac{-5}{2a^2}}$	**23.** $\frac{(3xy)^2(2x^4y^3)}{6x^8y}$ $\frac{18x^6y^5}{6x^8y}$ $3x^{-2}y^4 = \boxed{\frac{3y^4}{x^2}}$
	24. $\frac{(-6x^4y^6)^2}{(-4x^{-3}y^5)^3}$ $\frac{36x^8y^{12}}{-64x^{-9}y^{15}}$ $-\frac{9}{16}x^{17}y^{-3} = \boxed{-\frac{9x^{17}}{16y^3}}$	**25.** $\frac{(6bc^3)(3b^5c^2)}{(5b^5c^2)(2b^3c^6)}$ $\frac{18b^6c^5}{10b^8c^8}$ $\frac{9}{5}b^{-2}c^{-3} = \boxed{\frac{9}{5b^2c^3}}$
Challenge Problem! CHALLENGE AHEAD	Simplify completely: $\left(\frac{(2x^4y^{-3})^{-2}\cdot(2x^{-1}y^{-2})^4}{2x^{-7}y^{-3}}\right)^3$ $\left(\frac{\frac{1}{4}x^{-8}y^6\cdot 16x^{-4}y^{-8}}{2x^{-7}y^{-3}}\right)^3$ $\left(\frac{4x^{-12}y^{-2}}{2x^{-7}y^{-3}}\right)^3 = (2x^{-5}y)^3 = \boxed{\frac{8y^3}{x^{15}}}$	

EXPONENT RULES

Graphic Organizer

Name	Rule	Examples
ADDING & SUBTRACTING MONOMIALS	**COMBINE LIKE TERMS!!!** (DO NOT CHANGE common variables and exponents!)	**1.** $9x^2y - 10x^2y =$ $-x^2y$ **2.** Subtract $6w$ from $8w$. $2w$
PRODUCT RULE	$x^a \cdot x^b =$ x^{a+b}	**1.** $h^2 \cdot h^6 =$ h^8 **2.** $(-2a^2b)\cdot(7a^3b) =$ $-14a^5b^2$
POWER RULE	$(x^a)^b =$ x^{ab}	**1.** $(x^2)^3 =$ x^6 **2.** $(-2m^5)^2 \cdot m^3 =$ $4m^{13}$
QUOTIENT RULE	$\frac{x^a}{x^b} =$ x^{a-b}	**1.** $\frac{27x^5}{42x} =$ $\frac{9}{14}x^4$ **2.** $\frac{(y^2)^2}{y^4} =$ 1
NEGATIVE EXPONENT RULE	$x^{-a} =$ $\frac{1}{x^a}$	**1.** $-5x^{-2} =$ $\frac{-5}{x^2}$ **2.** $\frac{4k^2}{8k^5} =$ $\frac{1}{2k^3}$
ZERO EXPONENT RULE	$x^0 =$ 1	**1.** $7x^0 =$ 7 **2.** $\frac{(w^4)^2}{w^8} =$ 1

POLYNOMIALS

- A **polynomial** is the sum or difference of many monomials.
- The **highest exponent** of a polynomial is called the **degree.**
- **Standard form:** Write exponents in descending order

CLASSIFYING
Polynomials

Classify by **DEGREE**		Classify by **NUMBER OF TERMS**	
0	Constant	1	monomial
1	linear	2	binomial
2	quadratic	3	trinomial
3	Cubic	4+	polynomial
4	quartic		
5	quintic		

ADDING & SUBTRACTING
Polynomials

Simplify. Write your answer in standard form and classify.

9. $(b^2+5b-11)+(8b^2-2b-7)$

$9b^2+3b-18$;

Quadratic Trinomial

10. $(3w^2+2w-6)-(9+3w^2+7w)$

$-5w-15$;

Linear Binomial

11. $(-3y^2+y^3-2)-(y-3y^3+4y^2)$

$4y^3-7y^2-y-2$;

Cubic Polynomial

12. $(5k^3-k^2+7)+(-4k+k^2+2)$

$5k^3-4k+9$;

Cubic Trinomial

MULTIPLYING
Polynomials

Simplify. Write your answer in standard form and classify.

13. $(x-3)(2x^2+6x-7)$

$= 2x^3+6x^2-7x-6x^2-18x+21$

$= \boxed{2x^3-25x+21;\ \text{Cubic trinomial}}$

14. $(4t-1)(5t+3)-2t(10t-9)$

$=20t^2+12t-5t-3-20t^2+18t$

$= \boxed{25t-3;\ \text{Linear Binomial}}$

15. $(4j-7k)^2$

$=(4j-7k)(4j-7k)$

$=16j^2-28jk-28jk+49k^2$

$= \boxed{16j^2-56jk+49k^2;\ \text{Quadratic Trinomial}}$

16. $(2v+3)(2v-3)(-3v+1)$

$=(4v^2-9)(-3v+1)$

$= \boxed{-12v^3+4v^2+27v-9;\ \text{Cubic Polynomial}}$

17. $(p^2+p-6)(p^2-2p+1)$

$=p^4-2p^3+p^2+p^3-2p+p-6p^2+12p-6$

$= \boxed{p^4-p^3-7p^2+13p-6;\ \text{Quartic Polynomial}}$

18. $(3x+1)^2(-x+4)^2$

$=(9x^2+6x+1)(x^2-8x+16)$

$=9x^4-72x^3+144x^2+6x^3-48x^2+96x+x^2-8x+16$

$= \boxed{9x^4-66x^3+97x^2+88x+16;\ \text{Quartic Polynomial}}$

19. A triangle has a base of $(3x+7)$ and a height of $(5x-1)$. A second triangle is drawn with a base that is tripled and a height that is doubled. Find the difference between the area of the original triangle and the area of the new triangle.

Original: $A=\frac{1}{2}(3x+7)(5x-1)=\frac{1}{2}(15x^2+32x-7)=7.5x^2+16x-3.5$

New: $A=\frac{1}{2}(9x+21)(10x-2)=\frac{1}{2}(90x^2+192x-42)=45x^2+96x-21$

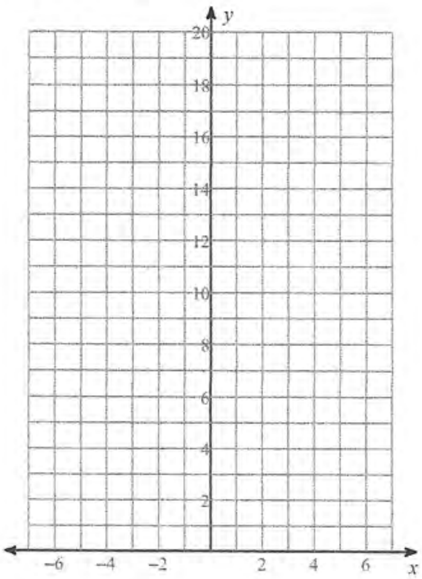
Difference: $(45x^2-96x-21)-(7.5x^2+16x-3.5)=\boxed{37.5x^2+80x-17.5}$

Precalculus Summer Packet Name____________________

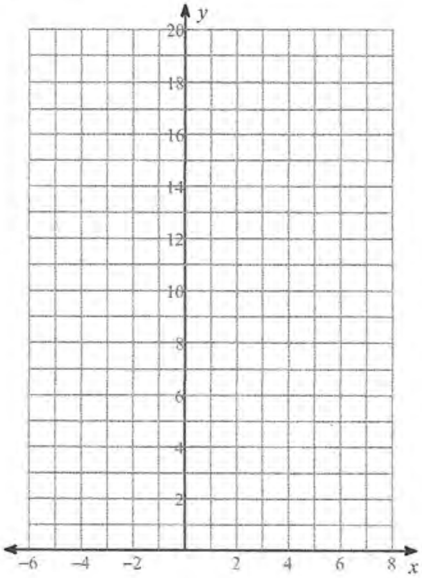
Exponential and Logarithmic Functions Week 2 (2 problems/day)

Sketch the graph of each function. State the equation of the horizontal asymptote. Clearly graph the asymptote.

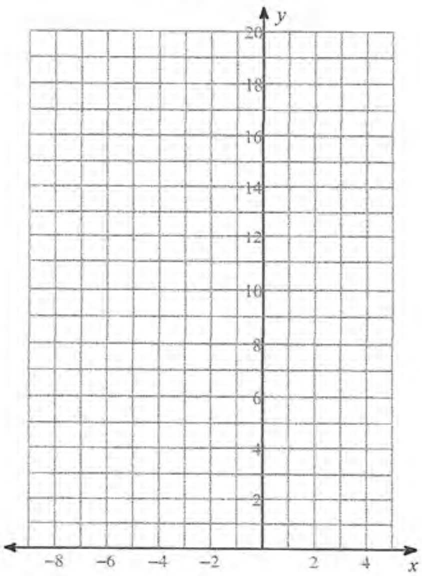
1) $y = 2 \cdot 3^{x}$



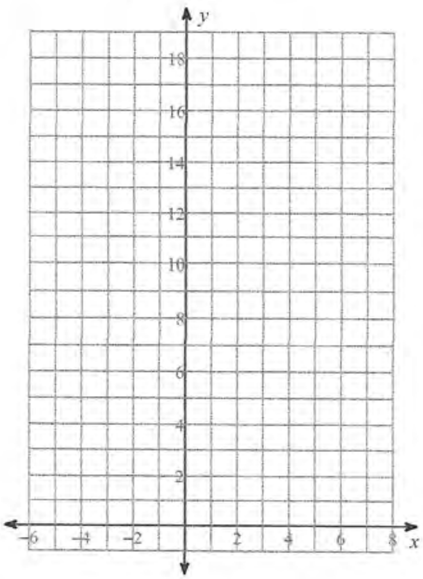
2) $y = 5 \cdot 2^{x-1}$



3) $f(x) = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^{x+2} + 2$



4) $f(x) = \frac{1}{3} \cdot \left(\frac{1}{5}\right)^{x-1} - 1$



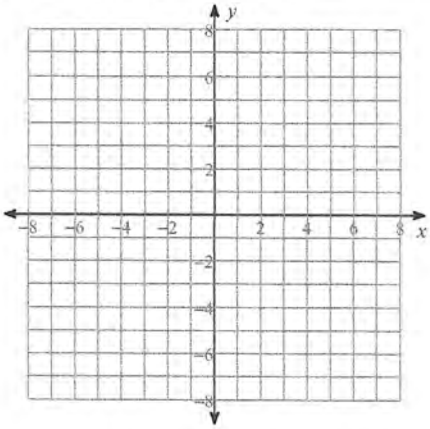
Week 2 (con't.)

Sketch the graph of each function. State the equation of the vertical asympotote. Clearly sketch the vertical asymptote. State the domain of the function in interval notation.

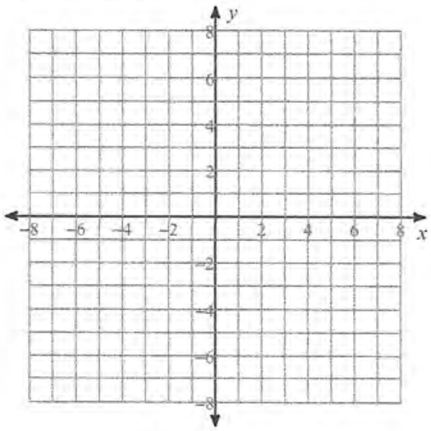
5) $f(x) = \log (x - 1) + 3$

Identify the domain and range of each function in interval notation. Then sketch the graph. State the equation of the vertical asymptote. Clearly sketch the vertical asymptote.

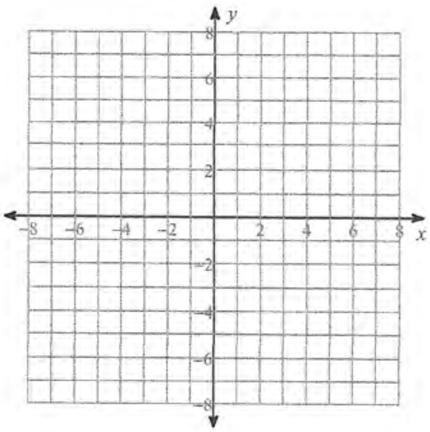
6) $y = \log_2 (x - 1) - 2$



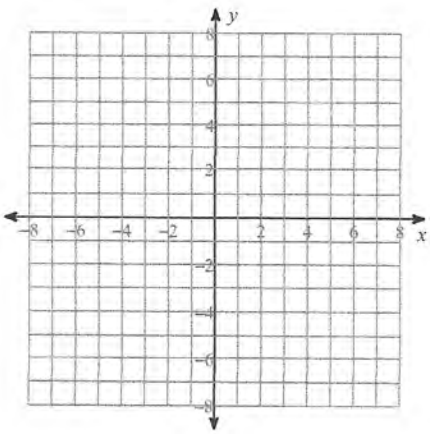
7) $y = \log_6 (x + 1) + 2$



8) $f(x) = \log_{\frac{1}{4}} (x + 2) + 2$



9) $f(x) = \log_4 (x + 1)$



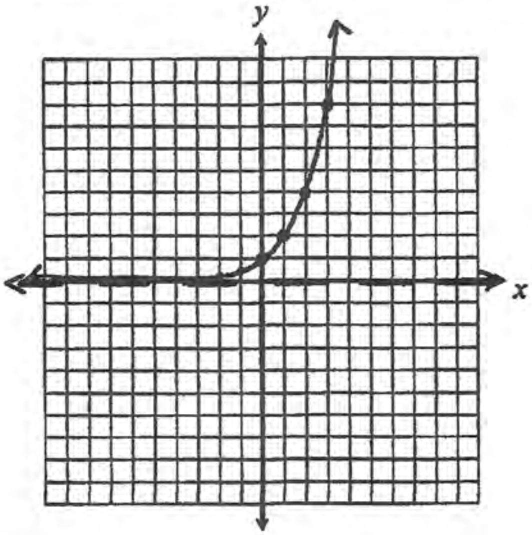
Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
EXPONENTIAL Parent Function $\boxed{f(x) = b^x}$	• If $b > 1$, the function is an exponential growth and is increasing. • If $b < 1$, the function is an exponential decay and is decreasing.
ASYMPTOTE	A line which the graph gets close to but never crosses

Directions: Classify as an exponential growth or decay, graph, then identify the key characteristics.

1. $f(x)=2^x$

x	y
-2	.25
-1	.5
0	1
1	2
2	4



Domain: $\mathbb{R}$

Range: $y>0$

End Behavior:

As $x \to \infty$, $f(x) \to \infty$

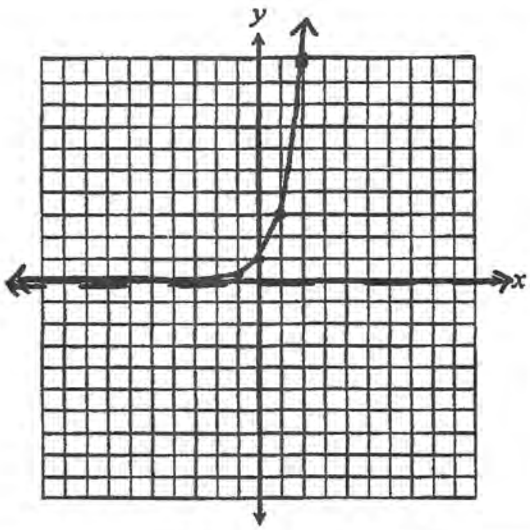
As $x \to -\infty$, $f(x) \to 0$

y-intercept: $(0,1)$

Asymptote: $y=0$

2. $f(x)=3^x$

x	y
-2	$.\overline{1}$
-1	$.\overline{3}$
0	1
1	3
2	9



Domain: $\mathbb{R}$

Range: $y>0$

End Behavior:

As $x \to \infty$, $f(x) \to \infty$

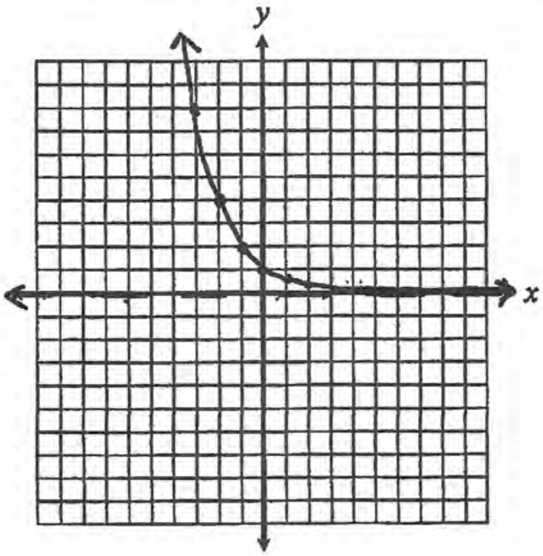
As $x \to -\infty$, $f(x) \to 0$

y-intercept: $(0,1)$

Asymptote: $y=0$

3. $f(x)=\left(\frac{1}{2}\right)^x$

X	y
-2	4
-1	2
0	1
1	.5
2	.25



Domain: ℝ

Range: $y > 0$

End Behavior:

As $x \to \infty$, $f(x) \to$ 0

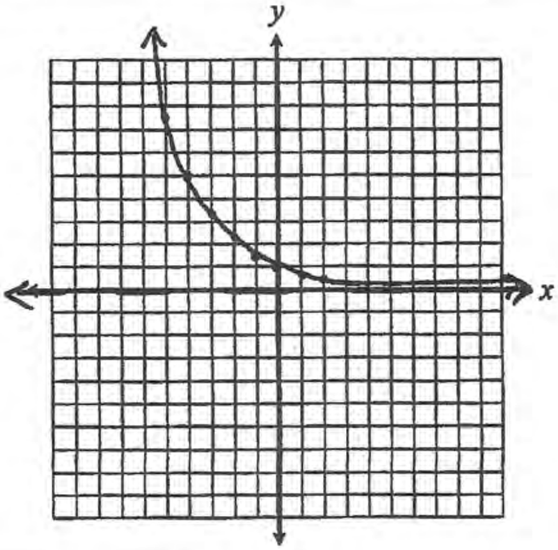
As $x \to -\infty$, $f(x) \to$ ∞

y-intercept: (0, 1)

Asymptote: $y = 0$

4. $f(x)=\left(\frac{2}{3}\right)^x$

X	Y
-2	2.25
-1	1.5
0	1
1	$0.\overline{6}$
2	$0.\overline{4}$



Domain: ℝ

Range: $y > 0$

End Behavior:

As $x \to \infty$, $f(x) \to$ 0

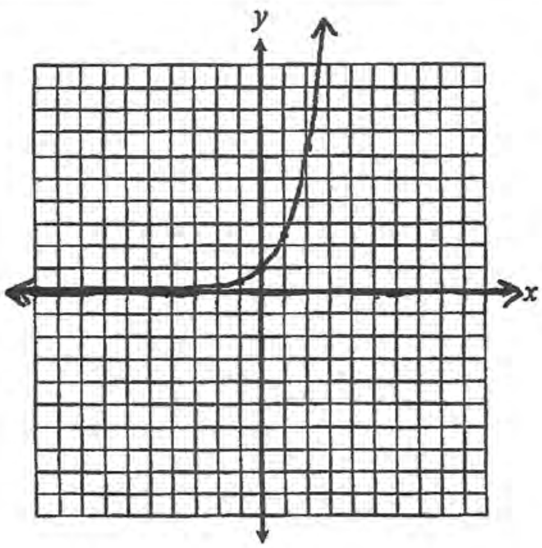
As $x \to -\infty$, $f(x) \to$ ∞

y-intercept: (0, 1)

Asymptote: $y = 0$

5. $f(x)=\left(\frac{5}{2}\right)^x$

X	Y
-2	0.16
-1	0.4
0	1
1	2.5
2	6.25



Domain: ℝ

Range: $y > 0$

End Behavior:

As $x \to \infty$, $f(x) \to$ ∞

As $x \to -\infty$, $f(x) \to$ 0

y-intercept: (0, 1)

Asymptote: $y = 0$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
TRANFORMATIONS *of Exponential Functions* $f(x) = a \cdot b^{x-h} + k$	• h is the horizontal shift. (+ shifts left, – shifts right) • k is the vertical shift. (+ shifts up, – shifts down) • If a is negative, the function is reflection across the x-axis • $\lvert a \rvert > 1$ represents a vertical stretch. • $0 < \lvert a \rvert < 1$ represents a vertical compression.

Directions: (a) Identify the parent function, and **(b)** describe the transformations.

1. $f(x) = 3^x + 5$

a) $f(x) = 3^x$

b) up 5

2. $f(x) = 2 \cdot \left(\frac{1}{4}\right)^{x-1}$

a) $f(x) = (1/4)^x$

b) vert. stretch x 2, right 1

3. $f(x) = -\left(\frac{4}{3}\right)^{x+2} + 7$

a) $f(x) = (4/3)^x$

b) reflect over x, left 2, up 7

4. $f(x) = \frac{1}{2} \cdot 5^{x-4} - 2$

a) $f(x) = 5^x$

b) vert. compression by 1/2, right 4, down 2

Directions: Graph each function and identify its key characteristics.

5. $f(x) = 2^{x+5}$

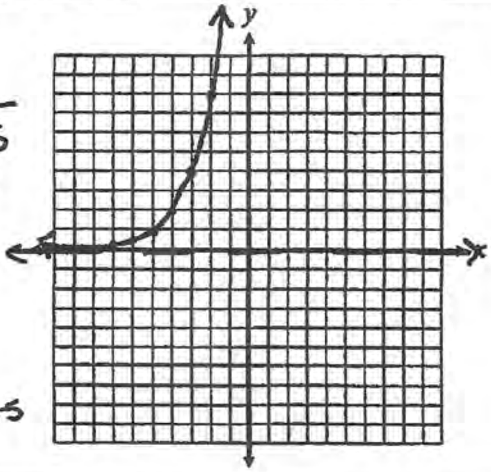
X	y
-2	0.25
-1	0.5
0	1
1	2
2	4

$f(x) = 2^x$

→

x	y
-7	0.25
-6	0.5
-5	1
-4	2
-3	4

$f(x) = 2^{x+5}$



Domain: $\mathbb{R}$

Range: $y > 0$

End Behavior:

As $x \to \infty$, $f(x) \to$ ∞

As $x \to -\infty$, $f(x) \to$ 0

y-intercept: $2^{0+5} = 32$ $(0, 32)$

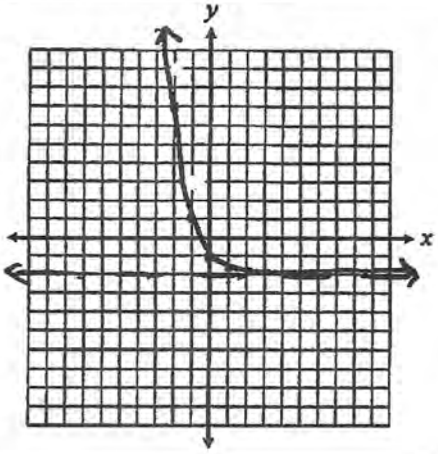
Asymptote: $y = 0$

6. $f(x)=\left(\frac{1}{3}\right)^{x}-2$

X	y
-2	9
-1	3
0	1
1	$0.\bar{3}$
2	$0.\bar{1}$

$\rightarrow$

X	y
-2	7
-1	1
0	-1
1	$-1.\bar{6}$
2	$-1.\bar{8}$



Domain: $\mathbb{R}$

Range: $y>-2$

End Behavior:

As $x\to\infty$, $f(x)\to$ -2

As $x\to-\infty$, $f(x)\to$ ∞

y-intercept: $(0,-1)$

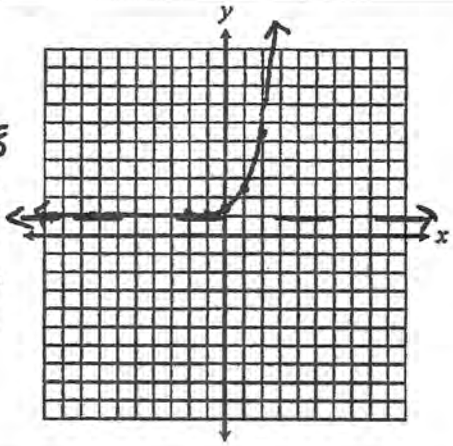
Asymptote: $y=-2$

7. $f(x)=\frac{1}{2}\cdot 3^{x}+1$

X	Y
-2	$0.\bar{1}$
-1	$0.\bar{3}$
0	1
1	3
2	9

$\rightarrow$

X	Y
-2	$1.0\bar{5}$
-1	$1.1\bar{6}$
0	1.5
1	2.5
2	5.5



Domain: $\mathbb{R}$

Range: $y>1$

End Behavior:

As $x\to\infty$, $f(x)\to$ ∞

As $x\to-\infty$, $f(x)\to$ 1

y-intercept: $(0, 1.5)$

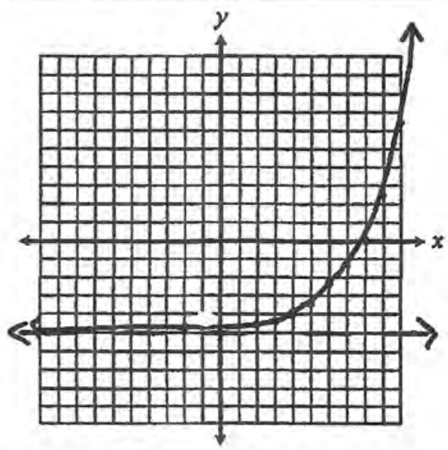
Asymptote: $y=1$

8. $f(x)=\left(\frac{3}{2}\right)^{x-4}-5$

X	Y
-2	$0.\bar{4}$
-1	$0.\bar{6}$
0	1
1	1.5
2	2.25

$\rightarrow$

X	Y
2	$-4.\bar{5}$
3	$-4.\bar{3}$
4	-4
5	-3.5
6	-2.75



Domain: $\mathbb{R}$

Range: $y>-5$

End Behavior:

As $x\to\infty$, $f(x)\to$ ∞

As $x\to-\infty$, $f(x)\to$ -5

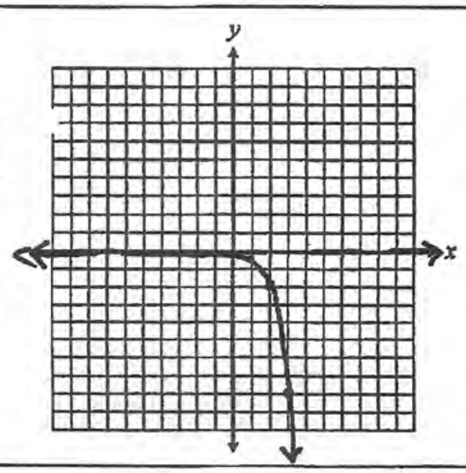
y-intercept: $(0, -4.802)$

Asymptote: $y=-5$

9. $f(x)=-2\cdot 4^{x-2}$

X	y
-2	.06
-1	.25
0	1
1	4
2	16

X	y
0	-.125
1	-0.5
2	-2
3	-8
4	-32



Domain: $\mathbb{R}$

Range: $y<0$

End Behavior:

As $x\to\infty$, $f(x)\to$ $-\infty$

As $x\to-\infty$, $f(x)\to$ 0

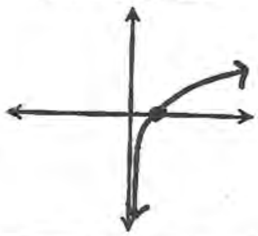
y-intercept: $(0, -0.125)$

Asymptote: $y=0$

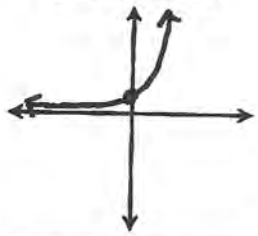
Name:	Date:
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples
LOGARITHMIC *Parent Function* $\boxed{f(x) = \log x}$	A logarithmic function is the **inverse** of an exponential function. Using your graphing calculator, sketch the following graphs:

$\boxed{f(x) = \log x}$



$\boxed{f(x) = 10^x}$



Because you can only graph base 10 logs on your calculator, you will need to use the inverse exponential function, then invert the values from the table to graph the logarithmic function.

Directions: Graph each function and identify its key characteristics.

1. $f(x) = \log_2 x$

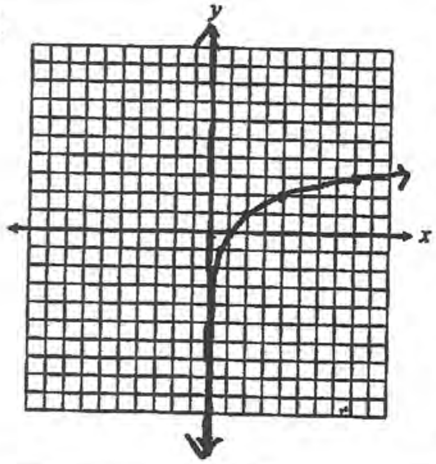
$f(x) = 2^x$

x	y
-2	.25
-1	.5
0	1
1	2
2	4

Inverse

$f(x) = \log_2 x$

x	y
.25	-2
.5	-1
1	0
2	1
4	2



Domain: x > 0

Range: ℝ

End Behavior:

As $x \to \infty$, $f(x) \to \infty$

As $x \to 0$, $f(x) \to -\infty$

x-intercept: (1, 0)

Asymptote: x = 0

2. $f(x) = \log_{\frac{1}{3}} x$

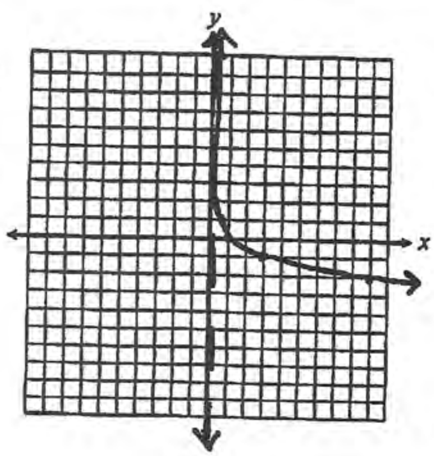
$f(x) = \frac{1}{3}^x$

x	y
-2	9
-1	3
0	1
1	$0.\bar{3}$
2	$0.\bar{1}$

inverse

$f(x) = \log_{1/3} x$

x	y
9	-2
3	-1
1	0
$0.\bar{3}$	1
$0.\bar{1}$	2



Domain: x > 0

Range: ℝ

End Behavior:

As $x \to 0$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to -\infty$

x-intercept: (1, 0)

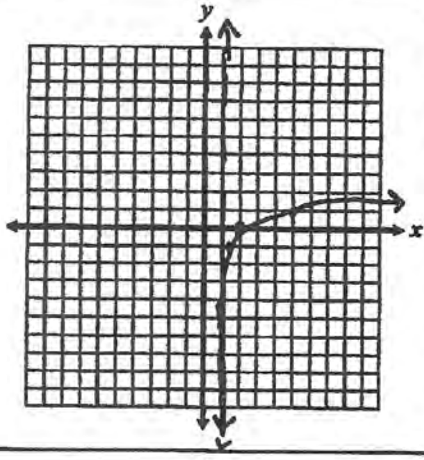
Asymptote: x = 0

3. $f(x)=\log_4(x-1)$

X	y
.06	-2
.25	-1
1	0
4	1
16	2

X	y
1.06	-2
1.25	-1
2	0
5	1
17	2

Right 1



Domain: X > 1

Range: ℝ

End Behavior:

As $x \to$ ∞, $f(x) \to \infty$

As $x \to$ 1, $f(x) \to -\infty$

x-intercept: (2,0)

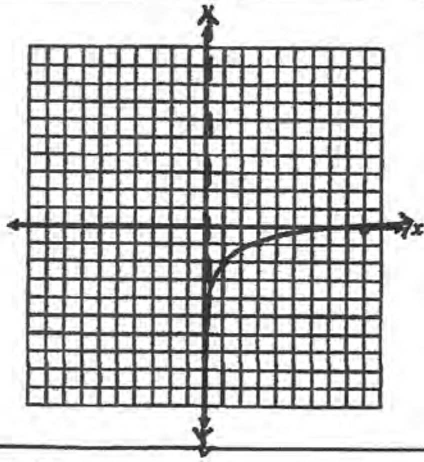
Asymptote: X = 1

4. $f(x)=\log_3 x-2$

X	y
$0.\overline{1}$	-2
$0.\overline{3}$	-1
1	0
3	1
9	2

X	y
$0.\overline{1}$	-4
$0.\overline{3}$	-3
1	-2
3	-1
9	0

down 2



Domain: X > 0

Range: ℝ

End Behavior:

As $x \to$ ∞, $f(x) \to \infty$

As $x \to$ 0, $f(x) \to -\infty$

x-intercept: (9,0)

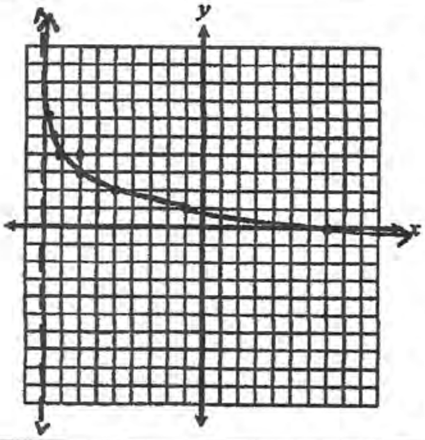
Asymptote: X = 0

5. $f(x)=\log_{\frac{1}{2}}(x+9)+4$

X	Y
4	-2
2	-1
1	0
.5	1
.25	2

X	Y
-5	2
-7	3
-8	4
-8.5	5
-8.75	6

Left 9, up 4



Domain: X > -9

Range: ℝ

End Behavior:

As $x \to$ -9, $f(x) \to \infty$

As $x \to$ ∞, $f(x) \to -\infty$

x-intercept: (7,0)

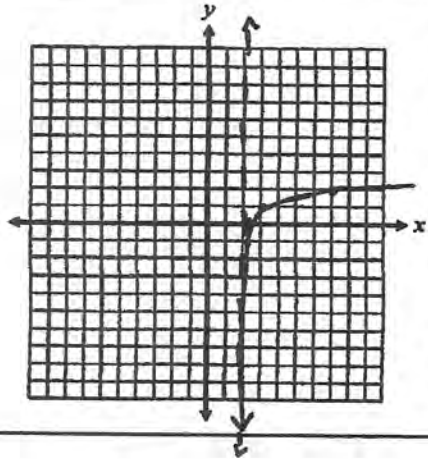
Asymptote: X = -9

6. $f(x)=\log_5(x-2)+1$

X	y
.04	-2
.2	-1
1	0
5	1
25	2

X	y
2.04	-1
2.2	0
3	1
7	2
27	3

Right 2, up 1



Domain: X > 2

Range: ℝ

End Behavior:

As $x \to$ ∞, $f(x) \to \infty$

As $x \to$ 2, $f(x) \to -\infty$

x-intercept: (2.2, 0)

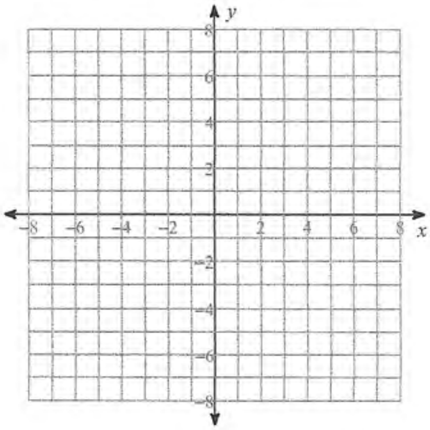
Asymptote: X = 2

Precalculus Summer Packet Name____________________

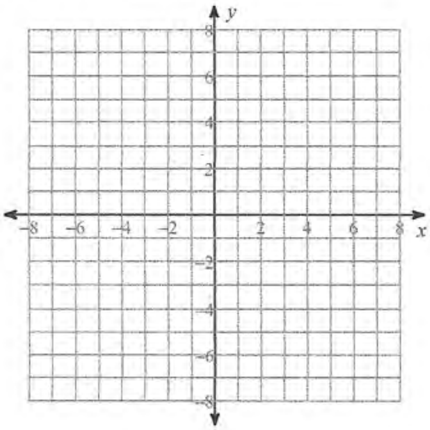
Polynomial and Rational Graphs Week 3 (1-2 problems/day)

Sketch the graph of each function.

1) $f(x) = -x^2 + 6x - 8$

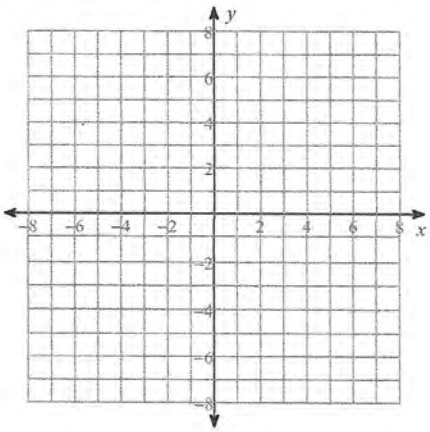


2) $f(x) = x^3 - 3x^2 + 4$

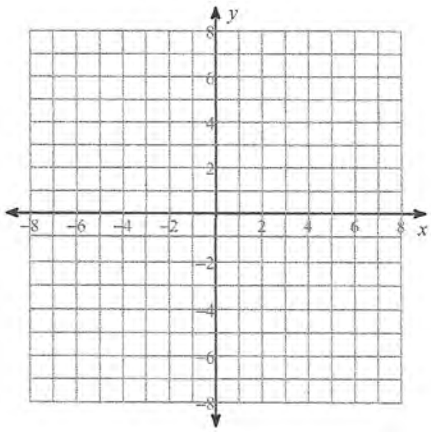


Identify the holes, vertical asymptotes, x-intercepts, horizontal asymptote, and domain of each. Then sketch the graph.

3) $f(x) = -\dfrac{1}{x+1}$

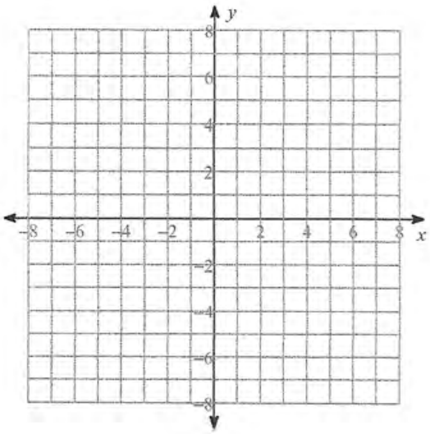


4) $f(x) = \dfrac{2}{x-3} + 2$

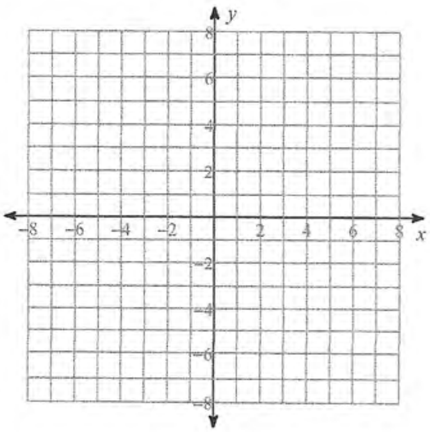


Week 3 (cont.)

5) $f(x) = \dfrac{x^2 - 9}{x^2 + 4x + 3}$



6) $f(x) = -\dfrac{2}{x^2 + 2x - 3}$



RATIONAL *Functions*

EQUATION FORM: $f(x) = \dfrac{p(x)}{q(x)}$

X-INTERCEPTS:
Set the numerator ($p(x)$) equal to zero.

VERTICAL ASYMPTOTES:
Set the denominator ($q(x)$) equal to zero.

HORIZONTAL ASYMPTOTES:

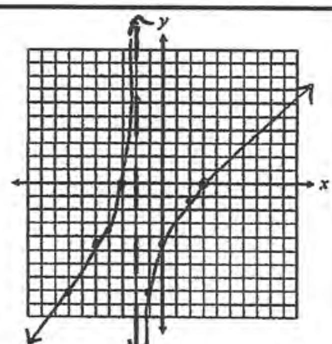
- If degree of p > degree of q: no horiz. asymptote
- If degree of p < degree of q: $y=0$
- If degree of p = degree of q: $y = \dfrac{\text{leading coeff } p(x)}{\text{leading coeff } q(x)}$

HOLES: (break in graph)
X-coord: Set common factor equal to zero
y-coord: substitute x-coord. into simplified function.

EXAMPLES

1 $f(x) = \dfrac{x^2-9}{x+2}$

$= \dfrac{(x+3)(x-3)}{x+2}$



x-int: $(-3,0)$, $(3,0)$ Holes: none

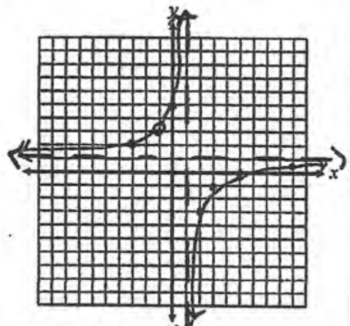
Vertical Asymptote: $x=-2$ D: $\{x \mid x \neq -2\}$

Horizontal Asymptote: none R: $\mathbb{R}$

2 $f(x) = \dfrac{x^2-4x-5}{x^2-1}$

$= \dfrac{(x-5)(x+1)}{(x-1)(x+1)}$

$= \dfrac{x-5}{x-1}$



x-int: $(5,0)$ Holes: $(-1,3)$

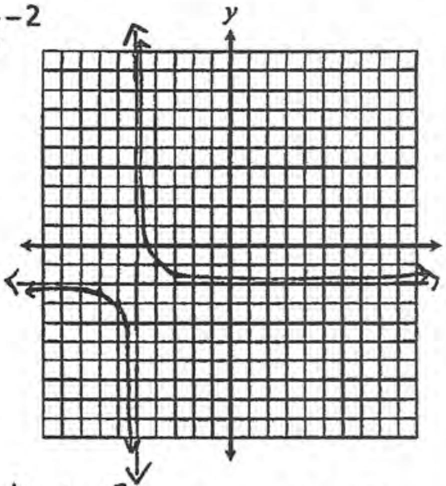
Vertical Asymptote: $x=1$ D: $\{x \mid x \neq -1, 1\}$

Horizontal Asymptote: $y=1$ R: $\{y \mid y \neq 1, 3\}$

GRAPHING RATIONAL FUNCTIONS *Practice!*

Directions: Graph each function and identify its key characteristics.

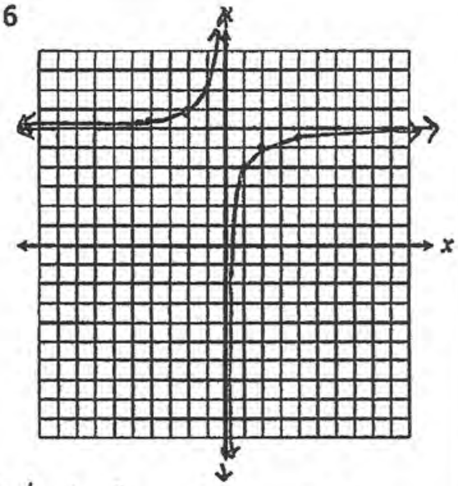
1. $f(x) = \frac{1}{x+5} - 2$



Domain: $\{x \mid x \neq -5\}$ **VA:** $x = -5$

Range: $\{y \mid y \neq -2\}$ **HA:** $y = -2$

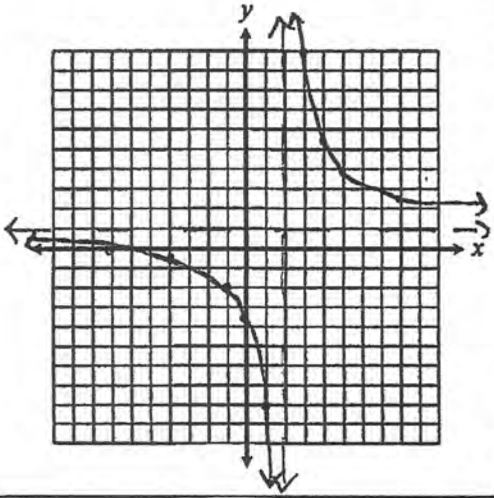
2. $f(x) = \frac{-2}{x} + 6$



Domain: $\{x \mid x \neq 0\}$ **VA:** $x = 0$

Range: $\{y \mid y \neq 6\}$ **HA:** $y = 6$

3. $f(x) = \frac{x+7}{x-2}$



x-int: $(-7, 0)$

VA: $x = 2$

HA: $y = 1$

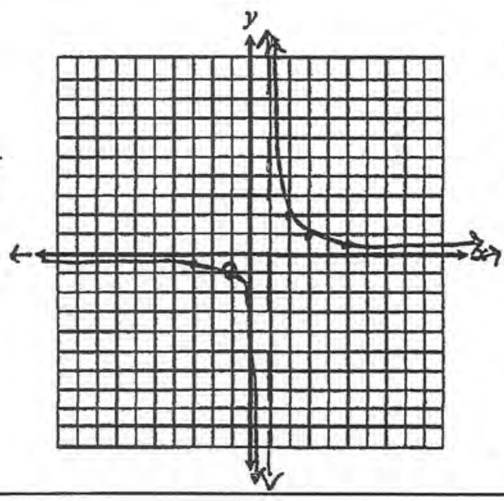
Hole: none

Domain: $\{x \mid x \neq 2\}$

Range: $\{y \mid y \neq 1\}$

4. $f(x) = \frac{2x+2}{x^2-1}$

$$= \frac{2(x+1)}{(x+1)(x-1)} = \frac{2}{x-1}$$



x-int: none

VA: $x = 1$

HA: $y = 0$

Hole: $(-1, -1)$

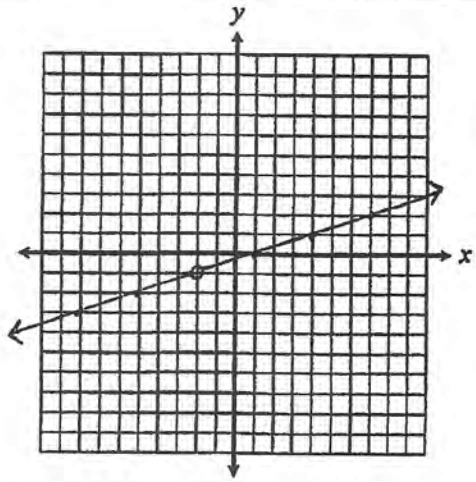
Domain: $\{x \mid x \neq -1, 1\}$

Range: $\{y \mid y \neq 0, -1\}$

5. $f(x) = \dfrac{x^2+x-2}{3x+6}$

$$= \frac{(x+2)(x-1)}{3(x+2)}$$

$$= \frac{x-1}{3}$$



x-int: $(1,0)$

VA: none

HA: none

Hole: $(-2,-1)$

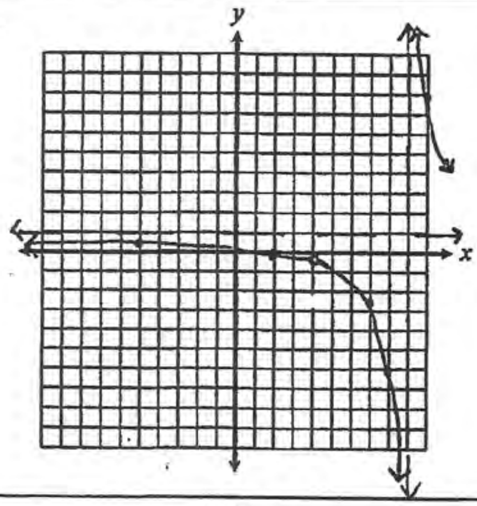
Domain: $\{x \mid x \neq -2\}$

Range: $\{y \mid y \neq -1\}$

6. $f(x) = \dfrac{x^2-6x+8}{x^2-13x+36}$

$$= \frac{(x-2)(x-4)}{(x-4)(x-9)}$$

$$= \frac{x-2}{x-9}$$



x-int: $(2,0)$

VA: $x=9$

HA: $y=1$

Hole: $(4, -2/5)$

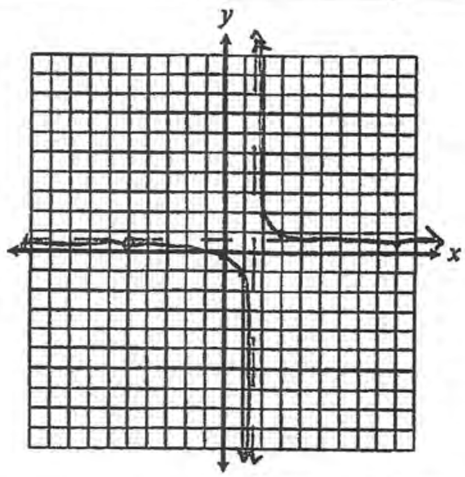
Domain: $\{x \mid x \neq 4, 9\}$

Range: $\{y \mid y \neq -2/5, 1\}$

7. $f(x) = \dfrac{x^2+5x}{2x^2+7x-15}$

$$= \frac{x(x+5)}{(2x-3)(x+5)}$$

$$= \frac{x}{2x-3}$$



x-int: $(0,0)$

VA: $x=1.5$

HA: $y=0.5$

Hole: $(-5, 5/13)$

Domain: $\{x \mid x \neq 1.5, -5)$

Range: $\{y \mid y \neq 0.5, 5/13\}$

8. Write a **reciprocal function** with asymptotes $x = 4$ and $y = -9$.

$$f(x) = \frac{1}{x-4} - 9$$

9. Write a **rational function** with an *x*-intercept at (1, 0) and vertical asymptotes at $x = -3$ and $x = -5$. What is the horizontal asymptote?

$$f(x) = \frac{x-1}{(x+3)(x+5)}$$

HA: $y=0$

$$f(x) = \frac{x-1}{x^2+8x+15}$$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
RATIONAL FUNCTIONS	A function of the form: $\boxed{f(x) = \frac{p(x)}{q(x)}}$ — $p(x)$ and $q(x)$ are polynomials + $q(x) \neq 0$!
STEPS TO GRAPH	① **SIMPLIFY** the function.
	② Find the **x-intercept(s)** by setting the **numerator** equal to 0.
	③ Find the **vertical asymptote(s)** by setting the **denominator** equal to 0.
	④ Find the **horizontal asymptote** using the rules below.
WHAT IS A HOLE?	A hole is a point (x, y) at which there is a **break** in the graph. A hole occurs when there is a **common factor** between the numerator and denominator. » **To find the x-coodinate:** Set the common factor equal to 0. » **To find the y-coordinate:** Substitute the x-coordinate into the simplified function.

CASE	HORIZONTAL ASYMPTOTE
degree of $p >$ degree of q	No horizontal asymptote
degree of $p <$ degree of q	x-axis / $y=0$
degree of $p =$ degree of q	$y = \frac{\text{leading coeff. of } p(x)}{\text{leading coeff. of } q(x)}$

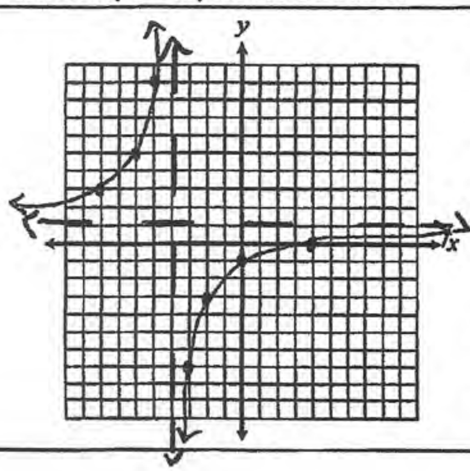
⑤ Identify any **holes** in the function.

Directions: Graph each function and identify its key characteristics.

1. $f(x) = \frac{x-4}{x+4}$

x-int: $x-4=0$
$x=4$

VA: $x+4=0$
$x=-4$



x-int: $(4,0)$

VA: $x=-4$

HA: $y=1$

Hole: none

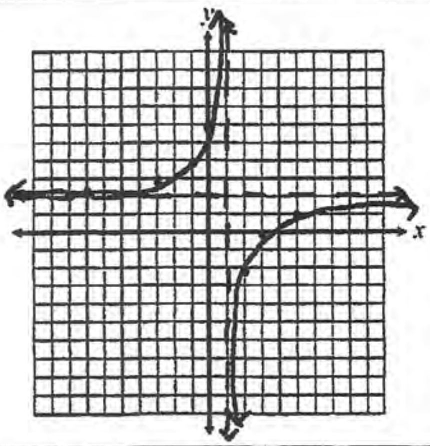
Domain: $\{x \mid x \neq -4\}$

Range: $\{y \mid y \neq 1\}$

2. $f(x)=\frac{2x-6}{x-1}$

X-int: $2x-6=0$
$x=3$

VA: $x-1=0$
$x=1$



x-int: (3,0)

VA: $x=1$

HA: $y=2$

Hole: none

Domain: $\{x \mid x \neq 1\}$

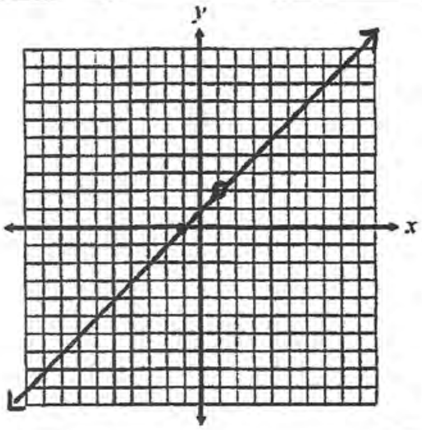
Range: $\{y \mid y \neq 2\}$

3. $f(x)=\frac{x^2-1}{x-1}$

$=\frac{(x-1)(x+1)}{x-1}$

$= x+1$

X-int: $x+1=0$
$x=1$



x-int: (-1,0)

VA: none

HA: none

Hole: (1, 2)

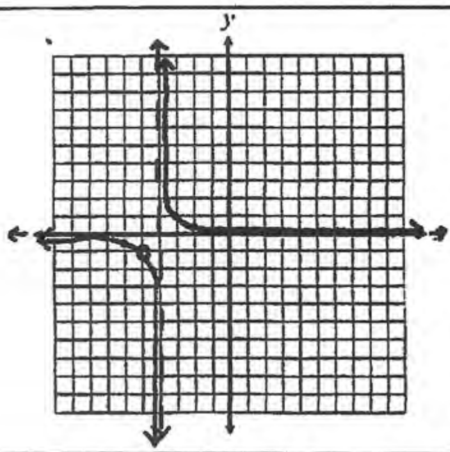
Domain: $\{x \mid x \neq 1\}$

Range: $\{y \mid y \neq 2\}$

4. $f(x)=\frac{x+5}{x^2+9x+20}$

$=\frac{x+5}{(x+4)(x+5)}$

$=\frac{1}{x+4}$



x-int: none

VA: $x=-4$

HA: $y=0$

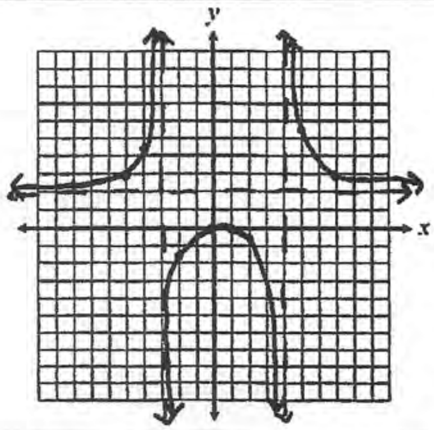
Hole: (-5, -1)

Domain: $\{x \mid x \neq -5, -4\}$

Range: $\{y \mid y \neq -1, 0\}$

5. $f(x)=\frac{2x^2-x-1}{x^2-x-12}$

$\frac{(2x+1)(x-1)}{(x-4)(x+3)}$



x-int: (-0.5, 0) and (1,0)

VA: $x=-3$, $x=4$

HA: $y=2$

Hole: none

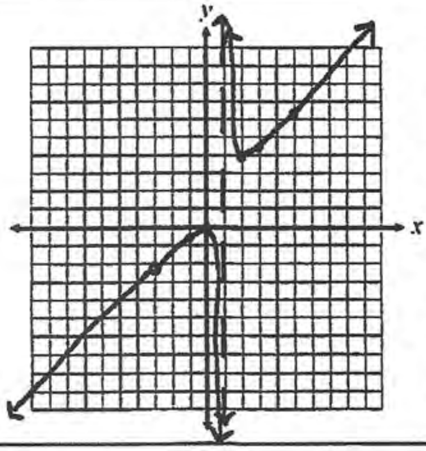
Domain: $\{x \mid x \neq -3, 4)$

Range: $\{y \mid y < 0.09$ or $y > 2\}$

6. $f(x) = \dfrac{x^3+3x^2}{x^2+2x-3}$

$= \dfrac{x^2(x+3)}{(x-1)(x+3)}$

$= \dfrac{x^2}{x-1}$



x-int: (0,0)

VA: $x = 1$

HA: none

Hole: $(-3, -9/4)$

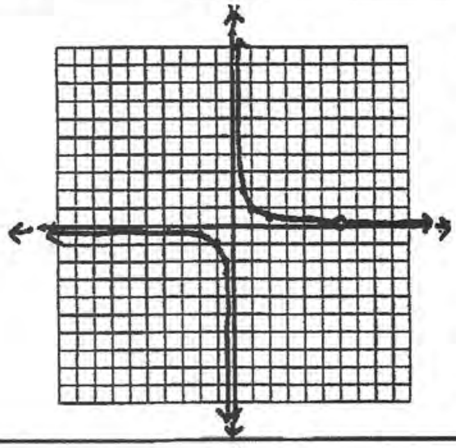
Domain: $\{x \mid x \neq -3, 1\}$

Range: $\{y \mid y \leq 0 \text{ or } y \geq 4\}$

7. $f(x) = \dfrac{x-6}{x^2-6x}$

$= \dfrac{x-6}{x(x-6)}$

$= \dfrac{1}{x}$



x-int: none

VA: $x=0$

HA: $y=0$

Hole: $(6, 1/6)$

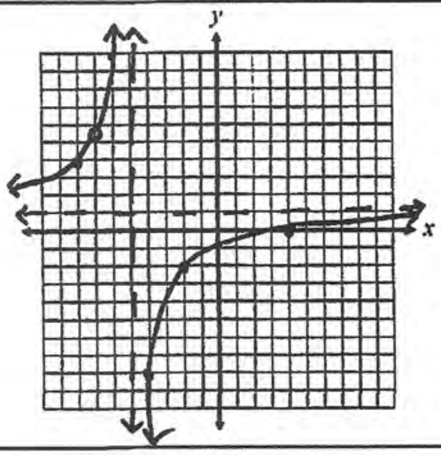
Domain: $\{x \mid x \neq 0, 6\}$

Range: $\{y \mid y \neq 0, 1/6\}$

8. $f(x) = \dfrac{x^2+3x-28}{x^2+12x+35}$

$= \dfrac{(x+7)(x-4)}{(x+7)(x+5)}$

$= \dfrac{x-4}{x+5}$



x-int: (4,0)

VA: $x = -5$

HA: $y = 1$

Hole: $(-7, 5.5)$

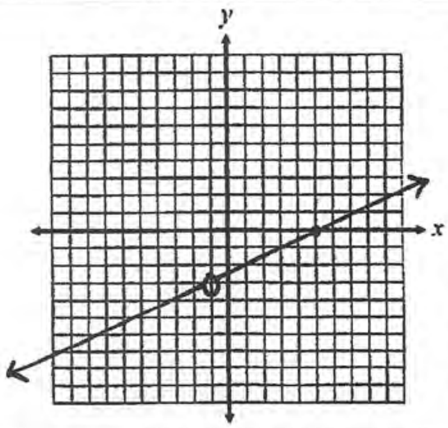
Domain: $\{x \mid x \neq -7, -5\}$

Range: $\{y \mid y \neq 1, 5.5\}$

9. $f(x) = \dfrac{x^2-4x-5}{2x+2}$

$= \dfrac{(x-5)(x+1)}{2(x+1)}$

$= \dfrac{x-5}{2}$



x-int: (5,0)

VA: none

HA: none

Hole: $(-1,-3)$

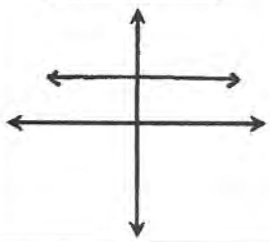
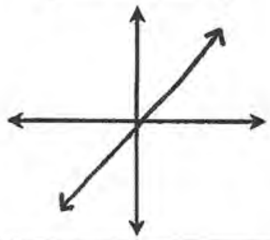
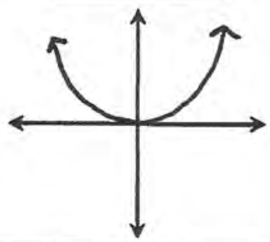
Domain: $\{x \mid x \neq -1\}$

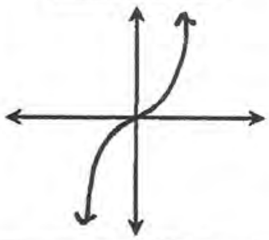
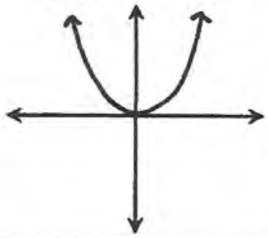
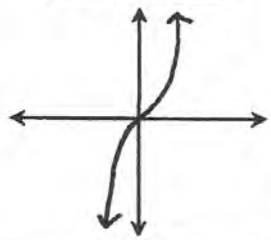
Range: $\{y \mid y \neq -3\}$

Name:	Date:
Topic:	Class:

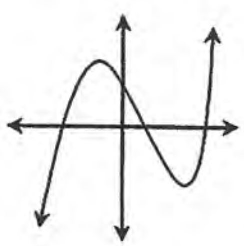
Main Ideas/Questions	Notes/Examples
POLYNOMIAL FUNCTIONS	A polynomial function is a function of the form $$f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots+a_2x^2+a_1x+a_0$$ where all coefficients are real numbers, all exponents are whole numbers, $a_n \neq 0$, and n is a positive integer. The **degree** of a polynomial function is the value of the greatest exponent.

Polynomial PARENT FUNCTIONS

CONSTANT (degree = 0)	LINEAR (degree = 1)	QUADRATIC (degree = 2)
$\boxed{f(x)=c}$	$\boxed{f(x)=x}$	$\boxed{f(x)=x^2}$
		

CUBIC (degree = 3)	QUARTIC (degree = 4)	QUINTIC (degree = 5)
$\boxed{f(x)=x^3}$	$\boxed{f(x)=x^4}$	$\boxed{f(x)=x^5}$
		

TURNING POINTS



- The point(s) at which a polynomial function **switches direction.**
- If the turning point is **higher** than any nearby point, it's called a relative maximum.
- If the turning point is **lower** than any nearby point, it's called a relative minimum.
- The maximum and minimum values are called extrema.
- Turning points define where a function is **increasing** or **decreasing.**

HOW TO FIND
Turning Points

Find the turning points of the function: $f(x) = x^3 - 6x^2 + 7x + 2$

Step 1: Graph the function. (Enter the function in $y =$, then hit **GRAPH**)

Step 2: Use the **CALC** menu to find the minimum and maximum values.

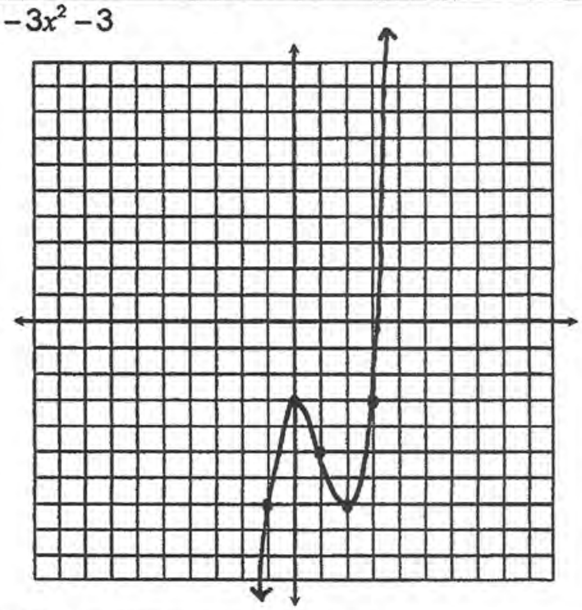
Step 3: Move the cursor to the left bound of the turning point. Hit **ENTER**, then move the cursor to the right bound of the turning point. Hit **ENTER** twice.

Rel. Maximum(s): (0.71, 4.30) **Rel. Minimum(s):** (3.29, -4.30)

Inc. Intervals: (-∞, 0.71), (3.29, ∞) **Dec. Intervals:** (0.71, 3.29)

Directions: Graph each polynomial function using a table. Then give the domain, range, turning points, increasing intervals, decreasing intervals, and end behavior.

1. $f(x) = x^3 - 3x^2 - 3$



Domain: {ℝ}

Range: {ℝ}

Rel. Maximum(s): (0, -3)

Rel. Minimum(s): (2, -7)

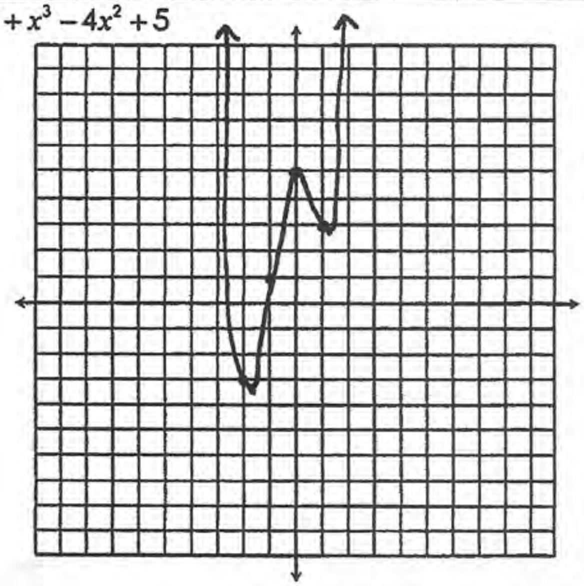
Inc. Intervals: (-∞, 0), (2, ∞)

Dec. Intervals: (0, 2)

End Behavior: As $x \to -\infty$, $f(x) \to$ -∞

As $x \to \infty$, $f(x) \to$ ∞

2. $f(x) = x^4 + x^3 - 4x^2 + 5$



Domain: {ℝ}

Range: {y | y ≥ -3.31}

Rel. Maximum(s): (0, 5)

Rel. Minimum(s): (-1.84, -3.31), (1.09, 2.95)

Inc. Intervals: (-1.84, 0), (1.09, ∞)

Dec. Intervals: (-∞, -1.84), (0, 1.09)

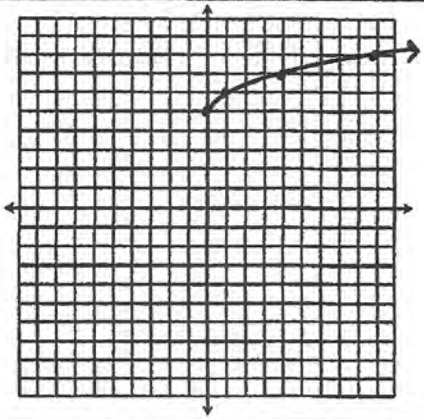
End Behavior: As $x \to -\infty$, $f(x) \to$ ∞

As $x \to \infty$, $f(x) \to$ ∞

GRAPHING SQUARE ROOT FUNCTIONS

Graph each function. Identify the key characteristics.

6. $f(x)=\sqrt{x}+5$



D: {x | x ≥ 0} **R:** {y | y ≥ 5}

Endpoint: (0,5)

End Behavior:

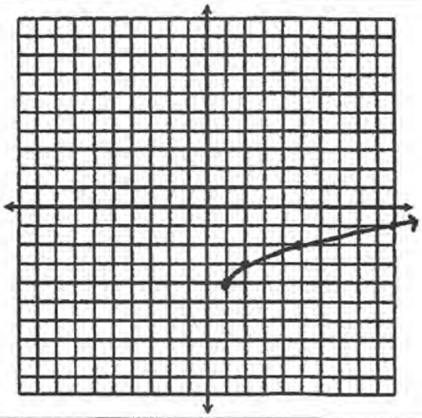
As $x \to$ ∞, $f(x) \to$ ∞

As $x \to$ 0, $f(x) \to$ 5

Increasing Interval(s): [0, ∞)

Decreasing Interval(s): N/A

7. $f(x)=\sqrt{x-1}-4$



D: {x | x ≥ 1} **R:** {y | y ≥ -4}

Endpoint: (1, -4)

End Behavior:

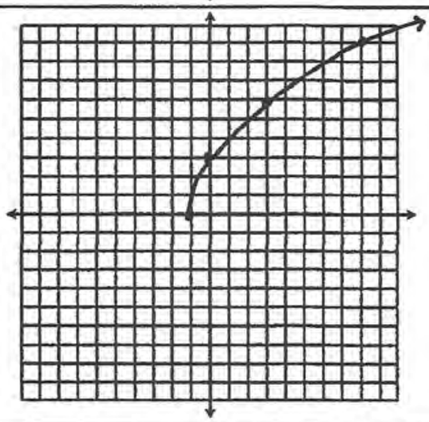
As $x \to$ ∞, $f(x) \to$ ∞

As $x \to$ 1, $f(x) \to$ -4

Increasing Interval(s): [1, ∞)

Decreasing Interval(s): N/A

8. $f(x)=3\sqrt{x+1}$



D: {x | x ≥ -1} **R:** {y | y ≥ 0}

Endpoint: (-1,0)

End Behavior:

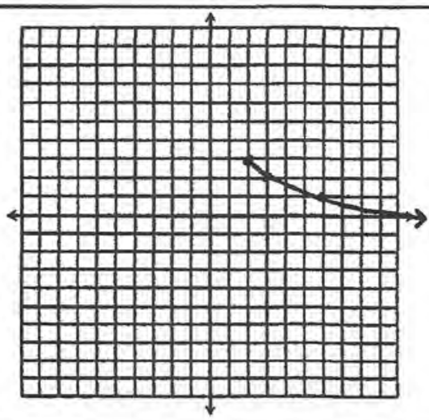
As $x \to$ ∞, $f(x) \to$ ∞

As $x \to$ -1, $f(x) \to$ 0

Increasing Interval(s): [-1, ∞)

Decreasing Interval(s): N/A

9. $f(x)=-\sqrt{x-2}+3$



D: {x | x ≥ 2} **R:** {y | y ≤ 3}

Endpoint: (2,3)

End Behavior:

As $x \to$ ∞, $f(x) \to$ -∞

As $x \to$ 2, $f(x) \to$ 3

Increasing Interval(s): N/A

Decreasing Interval(s): [2, ∞)

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

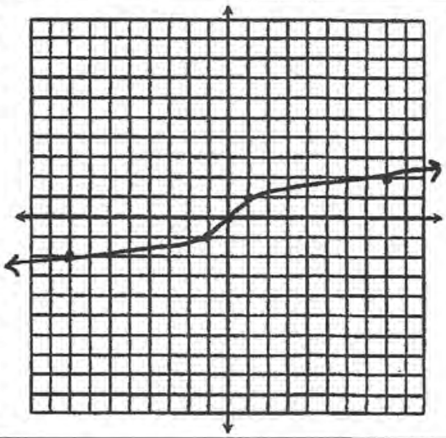
CUBE ROOT Function

Parent Function:

$\boxed{f(x) = \sqrt[3]{x}}$

The **cube root function** is another type of radical function.

Graph the parent function of the cube root function below and identify the key characteristics.



D: $\mathbb{R}$ **R:** $\mathbb{R}$

Point of Inflection: $(0, 0)$

End Behavior:

As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A

TRANFORMATIONS

$\boxed{f(x) = a\sqrt[3]{x-h} + k}$

- h is the horizontal shift. (+ shifts left, – shifts right)
- k is the vertical. shift. (+ shifts up, – shifts down)
- Point of Inflection: (h, k)
- If a is negative, the function is reflected across the X - axis
- $|a| > 1$ represents a vertical stretch.
- $0 < |a| < 1$ represents a vertical compression.

Describe the transformations of each function compared to the parent function.

1. $f(x) = \sqrt[3]{x} + 9$ Up 9	**2.** $f(x) = 5\sqrt[3]{x-2}$ Vert. stretch by 5, right 2
3. $f(x) = -\sqrt[3]{x+4} - 7$ Reflect across x-axis, Left 4, down 7	**4.** $f(x) = \frac{3}{4}\sqrt[3]{x-6} + 1$ Vert. compression by $\frac{3}{4}$, Right 6, up 1

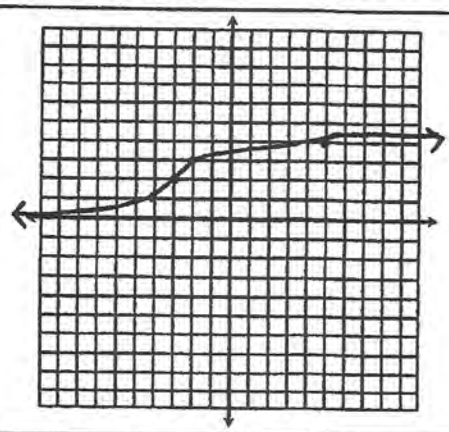
5. The cube root parent function is vertically stretched by a factor of 3, then translated so that its point of inflection is located at (-2, 3). Write an equation that represents this new function.

$f(x) = 3\sqrt[3]{x+2} + 3$

GRAPHING CUBE ROOT FUNCTIONS

Graph each function. Identify the key characteristics.

6. $f(x)=\sqrt[3]{x+3}+2$



D: $\mathbb{R}$ **R:** $\mathbb{R}$

Point of Inflection: $(-3, 2)$

End Behavior:

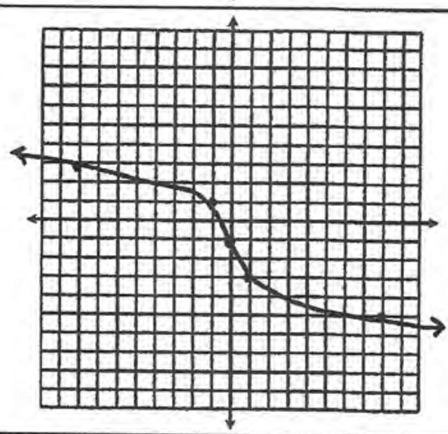
As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A

7. $f(x)=-2\sqrt[3]{x}-1$



D: $\mathbb{R}$ **R:** $\mathbb{R}$

Point of Inflection: $(0, -1)$

End Behavior:

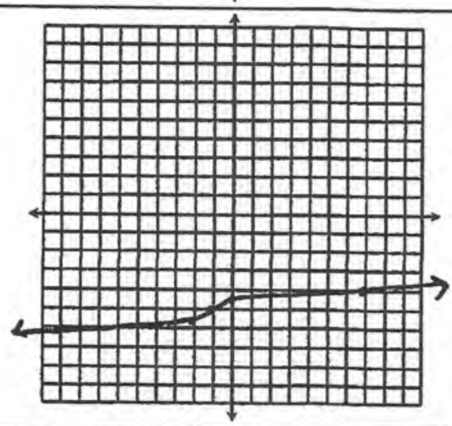
As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

Increasing Interval(s): N/A

Decreasing Interval(s): $(-\infty, \infty)$

8. $f(x)=\frac{1}{2}\sqrt[3]{x+1}-5$



D: $\mathbb{R}$ **R:** $\mathbb{R}$

Point of Inflection: $(-1, -5)$

End Behavior:

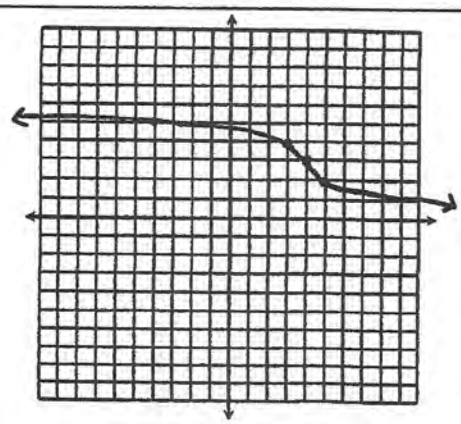
As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): N/A

9. $f(x)=-\sqrt[3]{x-4}+3$



D: $\mathbb{R}$ **R:** $\mathbb{R}$

Point of Inflection: $(4, 3)$

End Behavior:

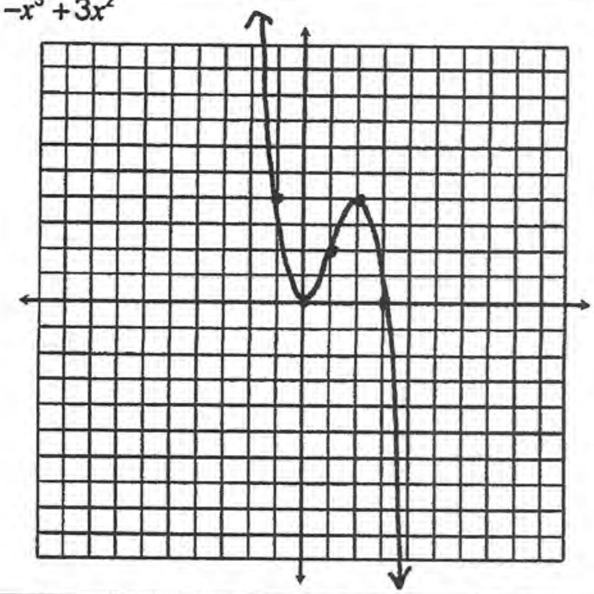
As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

Increasing Interval(s): N/A

Decreasing Interval(s): $(-\infty, \infty)$

3. $f(x) = -x^3 + 3x^2$



Domain: {ℝ}

Range: {ℝ}

Rel. Maximum(s): (2,4)

Rel. Minimum(s): (0,0)

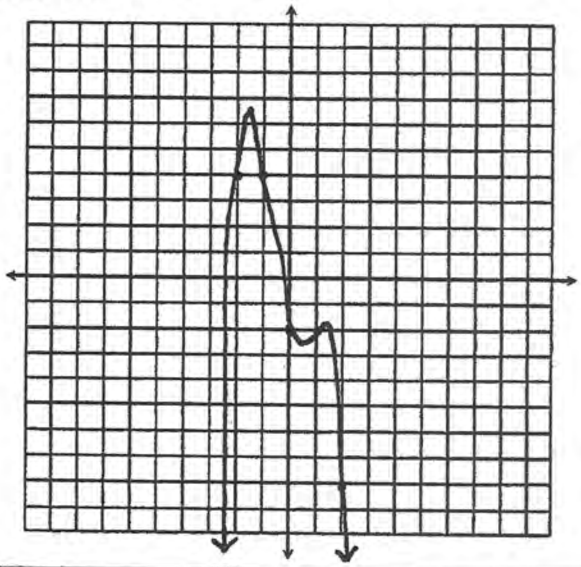
Inc. Intervals: (0,2)

Dec. Intervals: $(-\infty, 0)$, $(2, \infty)$

End Behavior: As $x \to -\infty$, $f(x) \to$ ∞

As $x \to \infty$, $f(x) \to$ $-\infty$

4. $f(x) = -x^4 + 4x^2 - 3x - 2$



Domain: {ℝ}

Range: $\{y \mid y \leq 6.5\}$

Rel. Maximum(s): (-1.57, 6.50), (1.16, -1.91)

Rel. Minimum(s): (0.41, -2.59)

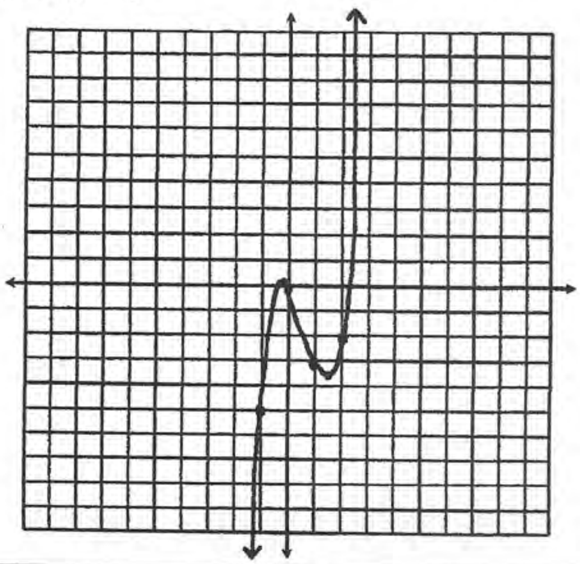
Inc. Intervals: $(-\infty, -1.57)$, $(0.41, 1.16)$

Dec. Intervals: $(-1.57, 0.41)$, $(1.16, \infty)$

End Behavior: As $x \to -\infty$, $f(x) \to$ $-\infty$

As $x \to \infty$, $f(x) \to$ $-\infty$

5. $f(x) = 2x^3 - 4x^2 - x$



Domain: {ℝ}

Range: {ℝ}

Rel. Maximum(s): (-0.12, 0.06)

Rel. Minimum(s): (1.45, -3.76)

Inc. Intervals: $(-\infty, -0.12)$, $(1.45, \infty)$

Dec. Intervals: $(-0.12, 1.45)$

End Behavior: As $x \to -\infty$, $f(x) \to$ $-\infty$

As $x \to \infty$, $f(x) \to$ ∞

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

END BEHAVIOR
Patterns

Using your graphing calculator, determine the end behavior of the following functions.

1. $f(x)=3x^2+6x-1$ As $x\to-\infty$, $f(x)\to$ ∞ As $x\to\infty$, $f(x)\to$ ∞	**2.** $f(x)=-x^2+x$ As $x\to-\infty$, $f(x)\to$ $-\infty$ As $x\to\infty$, $f(x)\to$ $-\infty$
3. $f(x)=2x^3-8x+2$ As $x\to-\infty$, $f(x)\to$ $-\infty$ As $x\to\infty$, $f(x)\to$ ∞	**4.** $f(x)=-x^3+5x+1$ As $x\to-\infty$, $f(x)\to$ ∞ As $x\to\infty$, $f(x)\to$ $-\infty$
5. $f(x)=x^4+2x^3-x-9$ As $x\to-\infty$, $f(x)\to$ ∞ As $x\to\infty$, $f(x)\to$ ∞	**6.** $f(x)=-x^4+3x^3-x^2$ As $x\to-\infty$, $f(x)\to$ $-\infty$ As $x\to\infty$, $f(x)\to$ $-\infty$

When written in standard form, the **leading coefficient** and **degree** of the polynomial function determines the end behavior of the function.

Draw sketches to help remember the end behavior patterns!

		LEADING COEFFICIENT	
		POSITIVE	NEGATIVE
DEGREE	EVEN	↑ ↑	↙ ↓
	ODD	↓ ↑	↑ ↓

EXAMPLES

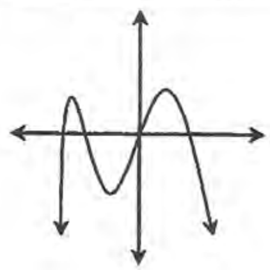
Identify the degree as even or odd, the leading coefficient as positive or negative, then sketch the end behavior with arrows.

FUNCTION	DEGREE	LEADING COEFFICIENT	END BEHAVIOR
7. $f(x)=-5x+1$	odd	negative	↑ ↓
8. $f(x)=12x^2-11x+31$	even	positive	↑ ↑
9. $f(x)=-x^3-x^2+8x+1$	odd	negative	↑ ↓

10. $f(x)=-3x^4+x^2-18$	even	negative	↓ ↓
11. $f(x)=9x^5+2x^4-x^3+9x^2-2$	odd	positive	↓ ↑
12. $f(x)=5x^4+2x^3-17x+1$	even	positive	↑ ↑
13. $f(x)=-x^7-5x^4+2x^2$	odd	negative	↑ ↓
14. $f(x)=-3x^2-14x+28$	even	negative	↓ ↓

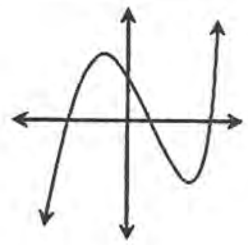
Determine the sign of the leading coefficient and whether the function has an even or odd degree.

15.



Negative ; Even degree

16.



Positive; odd degree

IDENTIFYING FUNCTIONS

Without graphing, check all functions with the described end behavior.

17. As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to -\infty$

- ☐ $f(x) = -x^3 + 10x^2 - 6x + 8$
- ☐ $f(x) = 3x^2 - 17x - 11$
- ☑ $f(x) = -4x^4 - x^3 + 8x^2 - 2x$
- ☐ $f(x) = 2x^5 + 8x^4 - 10x^3$
- ☐ $f(x) = -x^7 - 3x^2 + 9x - 14$
- ☑ $f(x) = -x^2 + 8x - 15$

18. As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to \infty$

- ☐ $f(x) = -x^2 - 14x + 32$
- ☑ $f(x) = x^3 + x^2 - 13x$
- ☑ $f(x) = 4x^7 - 8x^4 + 10x^2 + 2$
- ☐ $f(x) = -x^5 + 5x^2 - 13x - 4$
- ☐ $f(x) = -3x^4 + 2x^3 - x^2 + x$
- ☐ $f(x) = -x^3 - 12x^2 + 9x - 18$

19. As $x \to -\infty$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to -\infty$

- ☑ $f(x) = -2x^3 + 10x^2 - 7x$
- ☐ $f(x) = 5x^2 + x - 14$
- ☐ $f(x) = -x^6 + 4x^2 - 11x - 17$
- ☑ $f(x) = -x^5 - 4x^4 - x^3 + 2x - 1$
- ☐ $f(x) = -2x^4 - 7x^3 + x^2 - 6x$
- ☑ $f(x) = -x^3 - 12x + 20$

20. As $x \to -\infty$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to \infty$

- ☐ $f(x) = 4x^3 - 8x^2 - 2$
- ☑ $f(x) = 2x^6 + 5x^3 - 14x$
- ☐ $f(x) = -x^4 + x^3 - 6x^2 + x$
- ☑ $f(x) = x^2 - 9x + 36$
- ☑ $f(x) = 3x^4 + 7x^3 - 16x^2 + 3$
- ☐ $f(x) = -x^2 + 10x - 2$

Check for a **GCF** first!

FACTORING GUIDE

2 terms

DIFFERENCE OF SQUARES

$$a^2 - b^2$$

$(a+b)(a-b)$

$x^2-9 = (x+3)(x-3)$

$16x^4-25 = (4x^2+5)(4x^2-5)$

$48x^3-3x = 3x(16x^2-1)$
$= 3x(4x+1)(4x-1)$

SUM OF CUBES

$$a^3 + b^3$$

$(a+b)(a^2-ab+b^2)$

$x^3+27 = (x+3)(x^2-3x+9)$

$8x^6+1 = (2x^2+1)(4x^4-2x^2+1)$

$216x^3+125y^3$
$= (6x+5y)(36x^2-30xy+25y^2)$

DIFFERENCE OF CUBES

$$a^3 - b^3$$

$(a-b)(a^2+ab+b^2)$

$64-x^3 = (4-x)(16+4x+x^2)$

$27x^3-8 = (3x-2)(9x^2+6x+4)$

$512x^3-8 = 8(64x^3-1)$
$= 8(4x-1)(16x^2+4x+1)$

3 terms

TRINOMIALS $(a=1)$

Multiplies to "c" and adds to "b"

$x^2-6x+5 = (x-5)(x-1)$

$3x^2+3x-60 = 3(x^2+x-20)$
$= 3(x+5)(x-4)$

TRINOMIALS $(a>1)$

Multiplies to "a·c" and adds to "b"

$2x^2+15x-77 = (2x-7)(x+11)$

$6x^2-13x-28 = (3x+4)(2x-7)$

$9x^2+30x+25 = (3x+5)(3x+5)$
$= (3x+5)^2$

4 terms

TRY GROUPING!

Find the GCF of each part, then factor the binomials.

$x^3-2x^2+6x-12$
$= x^2(x-2)+6(x-2)$
$= (x^2+6)(x-2)$

$x^3+x^2-9x-9 = x^2(x+1)-9(x+1)$
$= (x^2-9)(x+1)$
$= (x+3)(x-3)(x+1)$

Precalculus Summer Packet Name________________________

Solving Equations – Week 4 (2 problems/day)

Solve each equation. Leave answers in exact, reduced form.

1) $-10\left|6n-2\right|=-80$

2) $\left|7m+3\right|+6=-75$

Solve each equation. Remember to check for extraneous solutions.

3) $\dfrac{5}{x}-\dfrac{x+6}{x^2-6x}=\dfrac{1}{x-6}$

4) $\dfrac{1}{3}=\dfrac{2n+4}{3n}+\dfrac{2}{3n}$

5) $5=\sqrt{3-11m}$

Solve each equation by factoring. Leave answers in exact, reduced form.

6) $k^2 + 16 = -8k$

7) $3m^2 - 12 = -16m$

Solve each equation with the quadratic formula. Leave answers in exact, reduced form.

8) $-n^2 + 7 = 5n$

Solve each equation by taking square roots. Leave answers in exact, reduced form.

9) $6n^2 - 1 = 155$

Solve each equation by completing the square. Leave answers in exact, reduced form.

10) $n^2 + 8n - 20 = 0$

Solving
ABSOLUTE VALUE EQUATIONS

Recall: To solve an absolute value equation-

① ISOLATE THE INEQUALITY ② CREATE TWO CASES ③ SOLVE EACH CASE

Solve each equation. Check for extraneous solutions.

9. $|-2-10w|=98$

$-2-10w=98$ $\quad$ $-2-10w=-98$

$-10w=100$ $\quad$ $-10w=-96$

$w=-10$ $\quad$ $w=9.6$

$\boxed{w=\{-10, 9.6\}}$

10. $|2b+4|-6=14$

$|2b+4|=20$

$2b+4=20$ $\quad$ $2b+4=-20$

$2b=16$ $\quad$ $2b=-24$

$b=8$ $\quad$ $b=-12$

$\boxed{b=\{-12, 8\}}$

11. $-4|7-a|=-36$

$|7-a|=9$

$7-a=9$ $\quad$ $7-a=-9$

$-a=2$ $\quad$ $-a=-16$

$a=-2$ $\quad$ $a=16$

$\boxed{a=\{-2, 16\}}$

12. $-43=2|6+8k|-9$

$-34=2|6+8k|$

$-17=|6+8k|$

$\boxed{\text{No Solution}}$

13. $1-10|9r+3|=-29$

$-10|9r+3|=-30$

$|9r+3|=3$

$9r+3=3$ $\quad$ $9r+3=-3$

$9r=0$ $\quad$ $9r=-6$

$r=0$ $\quad$ $r=-2/3$

$\boxed{r=\{-\frac{2}{3}, 0\}}$

14. $|5a+8|=3a-10$

$5a+8=3a-10$ $\quad$ $5a+8=-3a+10$

$2a=-18$ $\quad$ $8a=2$

$\cancel{a=-9}$ $\quad$ $\cancel{a=1/4}$

$\boxed{\text{No Solution}}$

15. $|6n-7|-5=3n$

$|6n-7|=3n+5$

$6n-7=3n+5$ $\quad$ $6n-7=-3n-5$

$3n=12$ $\quad$ $9n=2$

$n=4$ $\quad$ $n=2/9$

$\boxed{n=\{\frac{2}{9}, 4\}}$

16. $\frac{1}{5}|6x-1|=x+5$

$|6x-1|=5x+25$

$6x-1=5x+25$ $\quad$ $6x-1=-5x-25$

$x=26$ $\quad$ $11x=-24$

$x=\frac{-24}{11}$

$\boxed{x=\{-\frac{24}{11}, 26\}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
RATIONAL EXPRESSIONS	• A rational expression is a quotient of two polynomial expressions. • **Recall:** $\frac{x+y}{y+x}=1$ and $\frac{x-y}{y-x}=-1$

SIMPLIFYING Rational Expressions

(Factor & Simplify)

1. $\dfrac{64k^3-1}{4k^2-13k+3}$

$= \dfrac{(4k-1)(16k^2+4k+1)}{(4k-1)(k-3)}$

$\boxed{= \dfrac{16k^2+4k+1}{k-3}}$

2. $\dfrac{12n^2-48n}{288n^2-18n^4}$

$= \dfrac{12n(n-4)}{18n^2(16-n^2)}$

$= \dfrac{12n(n-4)}{18n^2(4+n)(4-n)} = \boxed{\dfrac{-2}{3n(4+n)}}$

MULTIPLYING & DIVIDING Rational Expressions

For Multiplication: Factor, then simplify.

3. $\dfrac{x^2-10x-24}{x^3+2x^2} \cdot \dfrac{5x^2-20x}{x^2-16x+48}$

$= \dfrac{(x-12)(x+2)}{x^2(x+2)} \cdot \dfrac{5x(x-4)}{(x-12)(x-4)}$

$= \dfrac{5x}{x^2} = \boxed{\dfrac{5}{x}}$

4. $\dfrac{5p^2+29p-6}{5p^2-26p+5} \cdot \dfrac{3p^2-15p}{36-p^2}$

$= \dfrac{(5p-1)(p+6)}{(5p-1)(p-5)} \cdot \dfrac{3p(p-5)}{(6-p)(6+p)}$

$= \boxed{\dfrac{3p}{6-p}}$

For Division: Multiply by the reciprocal.

5. $\dfrac{2s^2+9s+4}{1-4s^2} \div \dfrac{2s^2-s-1}{2s^2-3s+1}$

$= \dfrac{(2s+1)(s+4)}{(1+2s)(1-2s)} \cdot \dfrac{(2s-1)(s-1)}{(2s+1)(s-1)}$

$= \boxed{\dfrac{-(s+4)}{2s+1}}$

6. $\dfrac{k^4-16}{2k^2-k-6} \div \dfrac{4k^3+16k}{12k^2+18k}$

$= \dfrac{(k^2+4)(k+2)(k-2)}{(2k+3)(k-2)} \cdot \dfrac{6k(2k+3)}{4k(k^2+4)}$

$= \dfrac{6k(k+2)}{4k} = \boxed{\dfrac{3(k+2)}{2}}$

ADDING & SUBTRACTING Rational Expressions

Rewrite with common denominators, combine, then simplify.

7. $\dfrac{3}{4w}\cdot\dfrac{5}{5} - \dfrac{2}{5w}\cdot\dfrac{4}{4}$

$= \dfrac{15}{20w} - \dfrac{8}{20w}$

$= \boxed{\dfrac{7}{20w}}$

8. $\dfrac{2x-1}{x+4} - 3 \cdot \left(\dfrac{x+4}{x+4}\right)$

$\dfrac{2x-1}{x+4} - \dfrac{3x+12}{x+4}$

$= \boxed{\dfrac{-x-13}{x+4}}$

	9. $\frac{r+5}{r^2-25}+\frac{6}{r-5}\cdot\left(\frac{r+5}{r+5}\right)$ $=\frac{r+5}{r^2-25}+\frac{6r+30}{r^2-25}$ $=\frac{7r+35}{r^2-25}$ $=\frac{7(r+5)}{(r+5)(r-5)} = \boxed{\frac{7}{r-5}}$	10. $\frac{5a+6}{a^2-4}-\frac{1}{a+2}\cdot\left(\frac{a-2}{a-2}\right)$ $=\frac{5a+6}{a^2-4}-\frac{a-2}{a^2-4}$ $=\frac{4a+8}{a^2-4}$ $=\frac{4(a+2)}{(a+2)(a-2)} = \boxed{\frac{4}{a-2}}$
$\left(\frac{q-3}{q-3}\right)$	11. $\frac{q+1}{q-4}-\frac{q-1}{q^2-7q+12}$ $=\frac{q^2-2q-3}{q^2-7q+12}-\frac{q-1}{q^2-7q+12}$ $=\frac{q^2-3q-2}{q^2-7q+12}$ $=\boxed{\frac{q^2-3q-2}{(q-3)(q-4)}}$	$\left(\frac{x-1}{x-1}\right)\cdot\frac{3}{x+6}-\frac{2}{x-1}\cdot\left(\frac{x+6}{x+6}\right)$ $=\frac{3x-3}{x^2+5x-6}-\frac{2x+12}{x^2+5x-6}$ $=\frac{x-15}{x^2+5x-6}$ $=\boxed{\frac{x-15}{(x+6)(x-1)}}$
COMPLEX FRACTIONS Combine the numerator and denominator into a single fraction, multiply by the reciprocal, then simplify.	13. $\frac{\frac{4r^2}{4r^2}\cdot\frac{1}{10}-\frac{9}{40r^2}}{\frac{2r}{2r}\cdot\frac{1}{3}-\frac{1}{2r}\cdot\frac{3}{3}}$ $=\frac{4r^2-9}{40r^2}\cdot\frac{6r}{2r-3}$ $=\frac{(2r+3)(2r-3)}{40r^2}\cdot\frac{6r}{2r-3}$ $=\boxed{\frac{3(2r+3)}{20r}}$	
	14. $\frac{\frac{3}{3}\cdot\frac{y^3}{2}+\frac{y^2}{6}}{\frac{3y}{3y}\cdot 1+\frac{1}{3y}}$ $=\frac{3y^3+y^2}{6}\cdot\frac{3y}{3y+1}$ $=\frac{y^2(3y+1)}{6}\cdot\frac{3y}{3y+1} = \boxed{\frac{y^3}{2}}$	
	15. $\frac{\frac{2m^2}{2m^2}\cdot\frac{4m}{5n}+\frac{n^2}{10m^2}\cdot\frac{n}{n}}{\frac{2m}{2m}\cdot\frac{1}{n^2}+\frac{1}{2mn}\cdot\frac{n}{n}}$ $=\frac{8m^3+n^3}{10m^2n}\cdot\frac{2mn^2}{2m+n}$ $=\frac{(2m+n)(4m^2-2mn+n^2)}{10m^2n}\cdot\frac{2mn^2}{2m+n}$ $=\boxed{\frac{n(4m^2-2mn+n^2)}{5m}}$	

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
	Eliminate the radicals by squaring (or cubing, etc.) each side. Then solve the resulting equation. Always check for extraneous solutions!	
TYPE 1: Radical = Radical	1. $\sqrt{3x-20}^2 = \sqrt{24-x}^2$ $3x-20 = 24-x$ $4x = 44$ $\boxed{x=11}$	2. $\sqrt{2k+24}^2 = \sqrt{-8-2k}^2$ $2k+24 = -8-2k$ $4k = -32$ $\boxed{k=-8}$
	3. $\sqrt[3]{2-4a}^3 = \sqrt[3]{2a-1}^3$ $2-4a = 2a-1$ $3 = 6a$ $\boxed{\frac{1}{2} = a}$	4. $\sqrt[4]{7m-23}^4 = \sqrt[4]{3m+17}^4$ $7m-23 = 3m+17$ $4m = 40$ $\boxed{m=10}$
	ISOLATE the radical. Then eliminate the radical by squaring (or cubing, etc.) each side. Then solve the resulting equation. Always check for extraneous solutions!	
TYPE 2: Single Radical	5. $\sqrt{6y-17}^2 = 5^2$ $6y-17 = 25$ $6y = 42$ $\boxed{y=7}$	6. $4^3 = \sqrt[3]{1-7c}^3$ $64 = 1-7c$ $63 = -7c$ $\boxed{-9 = c}$
	7. $-9\sqrt{4g+13} = -45$ $\sqrt{4g+13}^2 = 5^2$ $4g+13 = 25$ $4g = 12$ $\boxed{g=3}$	8. $7 = 5 + \sqrt[4]{w-11}$ $2^4 = \sqrt[4]{w-11}^4$ $16 = w-11$ $\boxed{27=w}$

9. $2\sqrt[3]{2n+1}-9=-3$

$2\sqrt[3]{2n+1}=6$

$\left(\sqrt[3]{2n+1}\right)^3=3^3$

$2n+1=27$

$2n=26$

$\boxed{n=13}$

10. $\sqrt{18-7x}^{\,2}=x^2$

$18-7x=x^2$

$0=x^2+7x-18$

$0=(x+9)(x-2)$

$x\neq-9$ | $\boxed{x=2}$

11. $(p-7)^2=\sqrt{8p-63}^{\,2}$

$p^2-14p+49=8p-63$

$p^2-22p+112=0$

$(p-8)(p-14)=0$

$p=8$ | $p=14$

$\boxed{p=\{8,14\}}$

12. $\sqrt{2x+3}-x=2$

$\sqrt{2x+3}^{\,2}=(x+2)^2$

$2x+3=x^2+4x+4$

$0=x^2+2x+1$

$0=(x+1)(x+1)$

$x=-1$ | $x=-1$

$\boxed{x=-1}$

TYPE 3:
Rational Exponents

Solving an equation with a rational exponent is similar to solving an equation with a radical. In this case, isolate the power, then raise each side to the reciprocal of the power.

13. $(6p-23)^{\frac{1}{2}}+7=12$

$\left((6p-23)^{1/2}\right)^2=5^2$

$6p-23=25$

$6p=48$

$\boxed{p=8}$

14. $-12=-4(2z+9)^{\frac{1}{4}}$

$3^4=\left((2z+9)^{1/4}\right)^4$

$81=2z+9$

$72=2z$

$\boxed{z=36}$

15. $\left((5v-22)^{\frac{2}{3}}\right)^{3/2}=(4)^{3/2}$

$5v-22=\pm 8$

$5v-22=8$ | $5v-22=-8$

$5v=30$ | $5v=14$

$v=6$ | $v=\frac{14}{5}$

$\boxed{v=\left\{\frac{14}{5},6\right\}}$

16. $(x^2+6x)^{\frac{3}{4}}+7=15$

$\left((x^2+6x)^{3/4}\right)^{4/3}=8^{4/3}$

$x^2+6x=16$

$x^2+6x-16=0$

$(x+8)(x-2)=0$

$x=-8$ | $x=2$

$\boxed{x=\{-8,2\}}$

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples	
Solve by **FACTORING**	**Solve each equation by factoring.**	
	1. $y^2-13y-30=0$ $(y-15)(y+2)=0$ $y=15$ | $y=-2$ $\boxed{y=\{-2,15\}}$	**2.** $2x^2+6x=80$ $2x^2+6x-80=0$ $2(x^2+3x-40)=0$ $2(x+8)(x-5)=0$ $2\neq0$ | $x=-8$ | $x=5$ $\boxed{x=\{-8,5\}}$
	3. $4w^2=-10w$ $4w^2+10w=0$ $2w(2w+5)=0$ $w=0$ | $w=-\frac{5}{2}$ $\boxed{w=\{-\frac{5}{2},0\}}$	**4.** $5p^2=4+p$ $5p^2-p-4=0$ $(5p+4)(p-1)=0$ $p=-\frac{4}{5}$ | $p=1$ $\boxed{p=\{-\frac{4}{5},1\}}$
Solve by **SQUARE ROOTS**	**Solve each equation by square roots. Write all answers in simplest form.**	
	5. $16d^2-81=0$ $16d^2=81$ $d^2=\frac{81}{16}$ $d=\pm\sqrt{\frac{81}{16}}$ $\boxed{d=\pm\frac{9}{4}}$	**6.** $-6m^2=48$ $m^2=-8$ $m=\pm\sqrt{-8}$ $\boxed{m=\pm 2i\sqrt{2}}$
	7. $\frac{3}{2}x^2+11=59$ $\frac{3}{2}x^2=48$ $x^2=32$ $x=\pm\sqrt{32}$ $\boxed{x=\pm4\sqrt{2}}$	**8.** $10-7s^2=1$ $-7s^2=-9$ $s^2=\frac{9}{7}$ $s=\pm\sqrt{\frac{9}{7}}$ $s=\pm\frac{3}{\sqrt{7}}$ $\boxed{s=\pm\frac{3\sqrt{7}}{7}}$
Solve by **COMPLETING THE SQUARE**	❶	**REWRITE** as $ax^2+bx=c$
	❷	**DIVIDE** both sides by "a" so it becomes $x^2+bx=c$
	❸	**COMPLETE THE SQUARE** by taking half of b, square it, then **ADD THIS VALUE TO BOTH SIDES** of the equation.
	❹	**FACTOR** the perfect square trinomial.
	❺	Take the **SQUARE ROOT** of both sides. This will create two cases.
	❻	**SOLVE** both cases. **SIMPLIFY** all irrational and complex answers.

Solve by completing the square. Write all answers in simplest form.

9. $y^2+12y+20=0$ $y^2+12y=-20$ $y^2+12y+36=-20+36$ $(y+6)^2=16$ $y+6=\pm 4$ $y=-6\pm 4$ $\boxed{y=\{2,-10\}}$	10. $7b^2+14b-25=-4$ $7b^2+14b=21$ $b^2+2b=3$ $b^2+2b+1=3+1$ $(b+1)^2=4$ $b+1=\pm 2$ $b=-1\pm 2$ $\boxed{b=\{1,-3\}}$
11. $v^2-13v+8=-4$ $v^2-13v=-12$ $v^2-13v+\frac{169}{4}=-12+\frac{169}{4}$ $(v-\frac{13}{2})^2=\frac{121}{4}$ $v-\frac{13}{2}=\pm\frac{11}{2}$ $v=\frac{13}{2}\pm\frac{11}{2}$ $\boxed{v=\{1,12\}}$	12. $4a^2-20a+44=-5$ $4a^2-20a=-49$ $a^2-5a=-\frac{49}{4}$ $a^2-5a+\frac{25}{4}=-\frac{49}{4}+\frac{25}{4}$ $(a-\frac{5}{2})^2=\frac{-24}{4}$ $a-\frac{5}{2}=\pm\sqrt{-6}$ $\boxed{a=\left\{\frac{5}{2}\pm i\sqrt{6}\right\}}$

Solve by **THE QUADRATIC FORMULA**

Solve using the quadratic formula. Write all answers in simplest form.

13. $-k^2-8=10k$ $-k^2-10k-8=0$ $k=\frac{10\pm\sqrt{(-10)^2-4(-1)(-8)}}{2(-1)}$ $k=\frac{10\pm\sqrt{68}}{-2}$ $k=\frac{10\pm 2\sqrt{17}}{-2}$ $k=\boxed{\{-5\pm\sqrt{17}\}}$	14. $6x^2=3x-9$ $6x^2-3x+9=0$ $x=\frac{3\pm\sqrt{(-3)^2-4(6)(9)}}{2(6)}$ $x=\frac{3\pm\sqrt{9-216}}{12}$ $x=\frac{3\pm\sqrt{-207}}{12}$ $x=\frac{3\pm 3i\sqrt{23}}{12}$ $\boxed{x=\left\{\frac{1\pm i\sqrt{23}}{4}\right\}}$

APPLICATION

15. A rocket is launched upward with an initial speed of 325 ft/s. Its path is modeled by the equation $h=-16t^2+325t$, where h is the height of the rocket and t is the time in seconds since it launched. How long does the rocket take to reach the ground?

$-16t^2+325t=0$

$t=\frac{-325\pm\sqrt{325^2-4(-16)(0)}}{2(-16)}$

$t=\frac{-325\pm 325}{-32}$

$t=0, 20.3$

$\boxed{20.3 \text{ sec}}$

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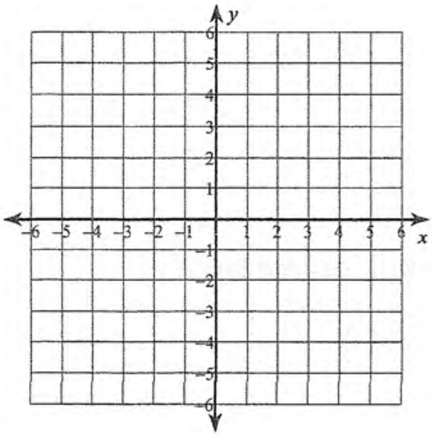
Graphing Absolute Value Equations - Week 5

3 problems/day

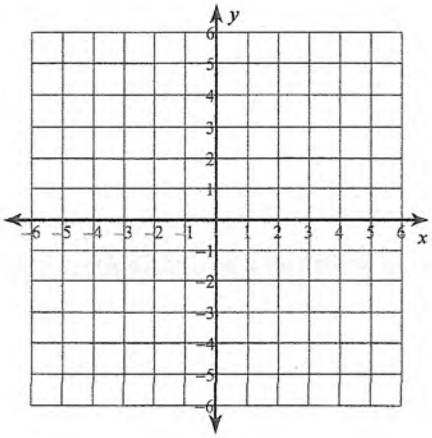
Date______________ Period____

Graph each equation.

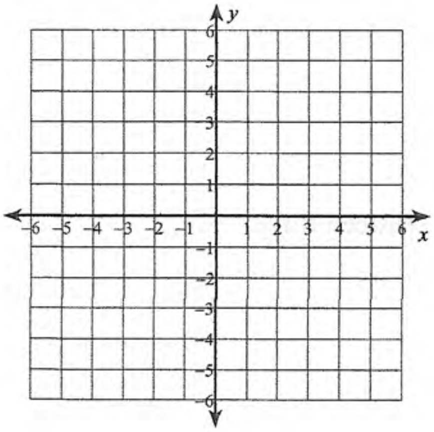
1) $y = |x - 1|$



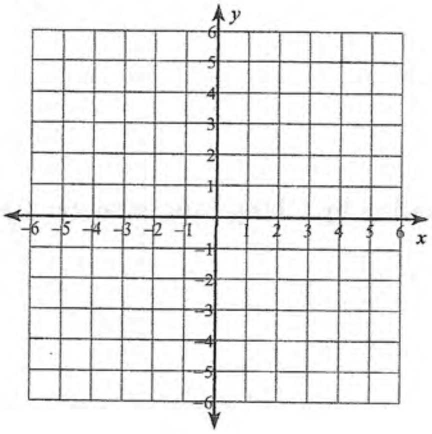
2) $y = |x + 4|$



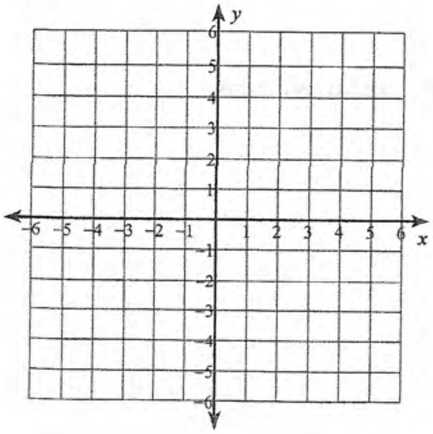
3) $y = |x - 2|$



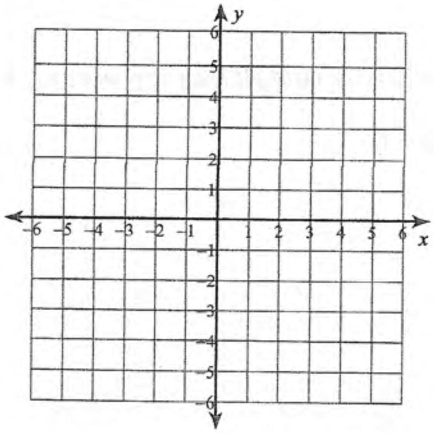
4) $y = 3|-3x - 3|$



5) $y = 2|-3x + 4| - 3$



6) $y = -3|-2x + 4| + 3$



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Graphing Radicals – Week 5 (Cont.)

Date________________ Period____

Identify the domain and range of each. 3 problems/day

1) $y = \sqrt{x-2} + 5$

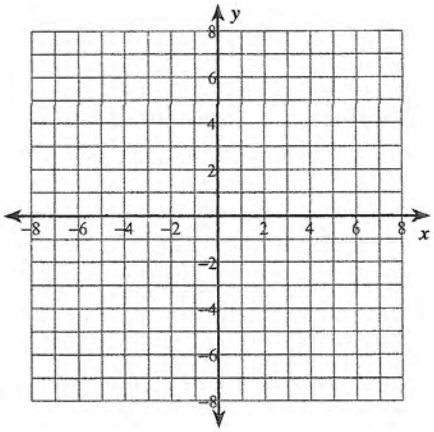
2) $y = \sqrt{x+2} - 3$

3) $y = \sqrt[3]{x+1} - 4$

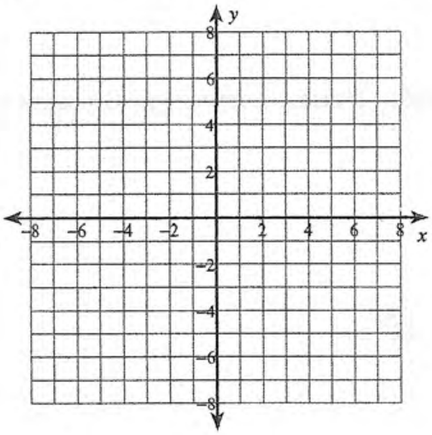
4) $y = \sqrt[3]{x-1} - 1$

Sketch the graph of each function.

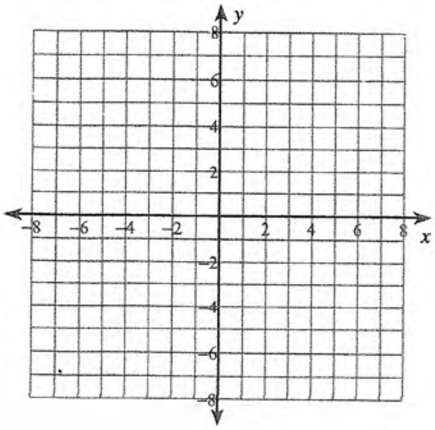
5) $y = \sqrt{x} + 5$



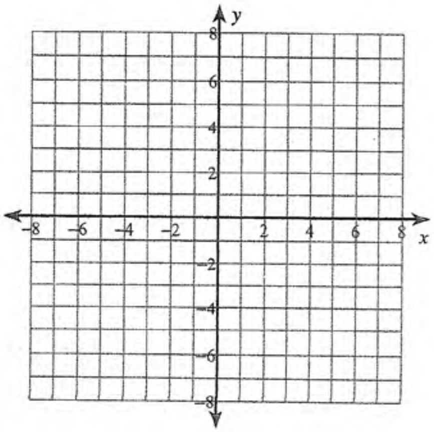
6) $y = \sqrt{x} - 2$



7) $y = \sqrt{x-4} - 2$



8) $y = -2 + \sqrt[3]{x}$



GRAPHING FUNCTIONS

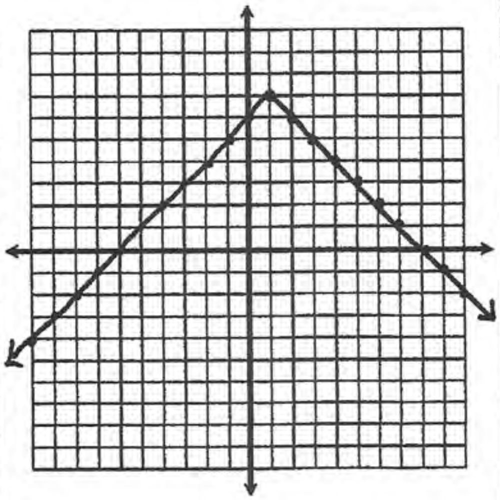
Directions: Identify the parent function and transformations from the parent function given each function. Then, graph the function using the transformations and identify its key characteristics.

1

$f(x) = -|x-1| + 7$

Parent Function: f(x) = |x|

Transformations: Reflect in x-axis, Translate right 1, up 7



Domain: ℝ	**Range:** {y | y ≤ 7}
x-intercept(s): (-6,0) + (8,0)	**y-intercept(s):** (0,6)

Extrema: (1,7) - Abs. Maximum

Increasing Interval(s): (-∞, 1)

Decreasing Interval(s): (1, ∞)

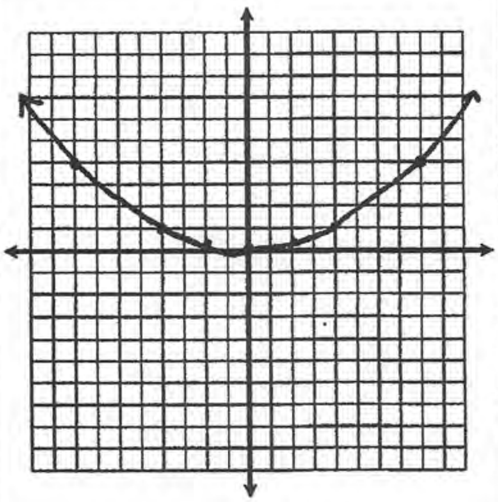
End Behavior:
As x → ∞, f(x) → -∞
As x → -∞, f(x) → -∞

2

$f(x) = \left(\frac{1}{4}x\right)^2$

Parent Function: $f(x) = x^2$

Transformations: Horizontal stretch by 4



Domain: ℝ	**Range:** {y | y ≥ 0}
x-intercept(s): (0,0)	**y-intercept(s):** (0,0)

Extrema: (0,0) - Abs. Minimum

Increasing Interval(s): (0, ∞)

Decreasing Interval(s): (-∞, 0)

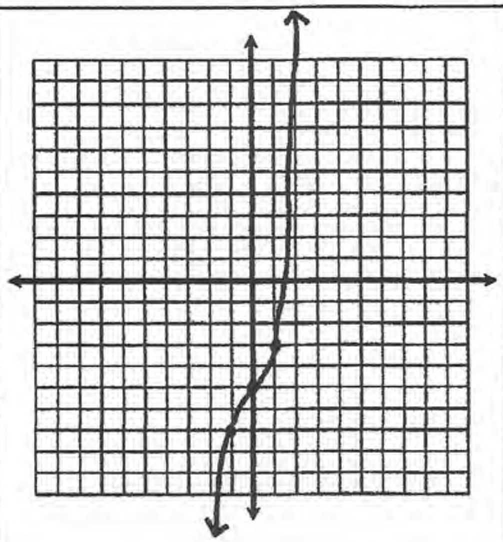
End Behavior:
As x → ∞, f(x) → ∞
As x → -∞, f(x) → ∞

3

$f(x) = 2x^3 - 5$

Parent Function: $f(x) = x^3$

Transformations: Vertical stretch by 2, Down 5



Domain: ℝ	**Range:** ℝ
x-intercept(s): (1.36, 0)	**y-intercept(s):** (0, -5)

Extrema: None

Increasing Interval(s): (-∞, ∞)

Decreasing Interval(s): None

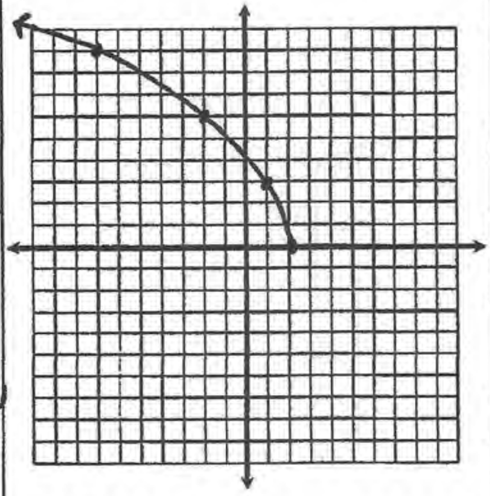
End Behavior:
As x → ∞, f(x) → ∞
As x → -∞, f(x) → -∞

4

$f(x) = 3\sqrt{-(x-2)}$

Parent Function: $y = \sqrt{x}$

Transformations: Vertical stretch by 3, Reflect in y-axis, Translate right 2



Domain: $\{x \mid x \le 2\}$

Range: $\{y \mid y \ge 0\}$

x-intercept(s): $(2, 0)$

y-intercept(s): $(0, 4.24)$

Extrema: $(2, 0)$ - Abs. Minimum

Increasing Interval(s): None

Decreasing Interval(s): $(-\infty, 2)$

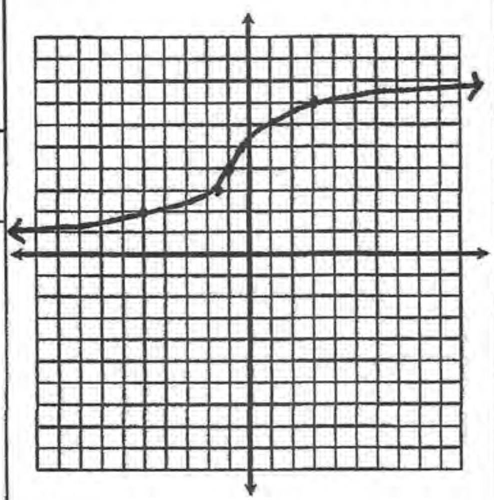
End Behavior:
As $x \to 2$, $f(x) \to 0$
As $x \to -\infty$, $f(x) \to \infty$

5

$f(x) = \sqrt[3]{2(x+1)} + 4$

Parent Function: $f(x) = \sqrt[3]{x}$

Transformations: Horizontal comp. by 1/2, Translate left 1, up 4



Domain: $\mathbb{R}$

Range: $\mathbb{R}$

x-intercept(s): $(-33, 0)$

y-intercept(s): $(0, 5.26)$

Extrema: None

Increasing Interval(s): $(-\infty, \infty)$

Decreasing Interval(s): None

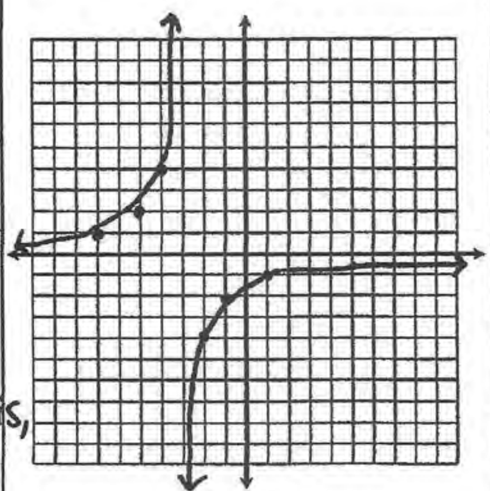
End Behavior:
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

6

$f(x) = \dfrac{-4}{x+3}$

Parent Function: $f(x) = \frac{1}{x}$

Transformations: Vertical stretch by 4, Reflect in x-axis, Translate left 3



Domain: $\{x \mid x \ne 3\}$

Range: $\{y \mid y \ne 0\}$

x-intercept(s): None

y-intercept(s): $(0, -1.\overline{3})$

Extrema: None

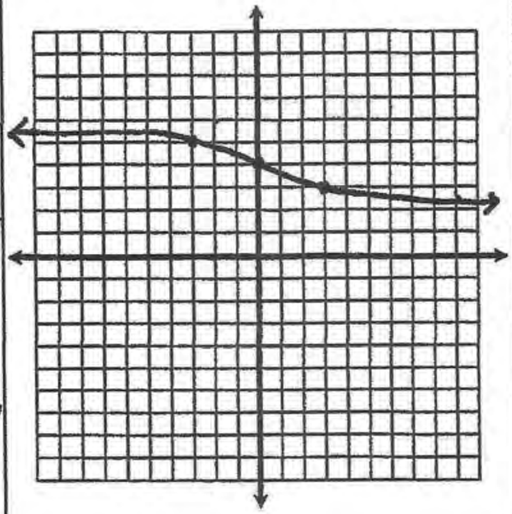
Increasing Interval(s): $(-\infty, -3)$, $(-3, \infty)$

Decreasing Interval(s): None

End Behavior:
As $x \to \infty$, $f(x) \to 0$
As $x \to -\infty$, $f(x) \to 0$

7

$f(x) = \sqrt[3]{-\frac{1}{3}x + 4}$



Domain: $\mathbb{R}$	**Range:** $\mathbb{R}$
***x*-intercept(s):** (192, 0)	***y*-intercept(s):** (0, 4)

Parent Function: $f(x) = \sqrt[3]{x}$

Transformations: Horiz. stretch by 3, Reflect in y-axis, Translate up 4

Extrema: None

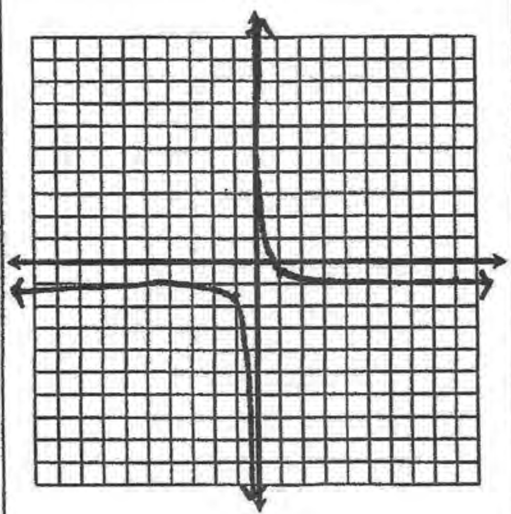
Increasing Interval(s): None

Decreasing Interval(s): $(-\infty, \infty)$

End Behavior:
As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to \infty$

8

$f(x) = \frac{1}{2x} - 1$



Domain: $\{x \mid x \neq 0$	**Range:** $\{y \mid y \neq -1\}$
***x*-intercept(s):** (0.5, 0)	***y*-intercept(s):** None

Parent Function: $f(x) = \frac{1}{x}$

Transformations: Vertical comp. by 1/2, Translate down 1

Extrema: None

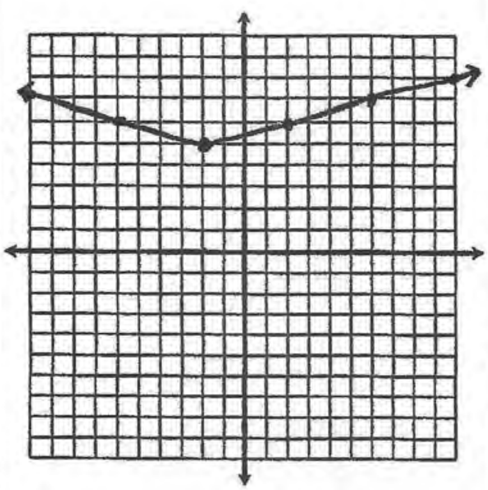
Increasing Interval(s): None

Decreasing Interval(s): $(-\infty, 0), (0, \infty)$

End Behavior:
As $x \to \infty$, $f(x) \to -1$
As $x \to -\infty$, $f(x) \to -1$

9

$f(x) = \frac{1}{4}|x + 2| + 5$



Domain: $\mathbb{R}$	**Range:** $\{y \mid y \geq 5\}$
***x*-intercept(s):** None	***y*-intercept(s):** (0, 5.5)

Parent Function: $f(x) = |x|$

Transformations: Vertical comp. by 1/4, Translate left 2, up 5

Extrema: (-2, 5) - Abs. Minimum

Increasing Interval(s): $(-2, \infty)$

Decreasing Interval(s): $(-\infty, -2)$

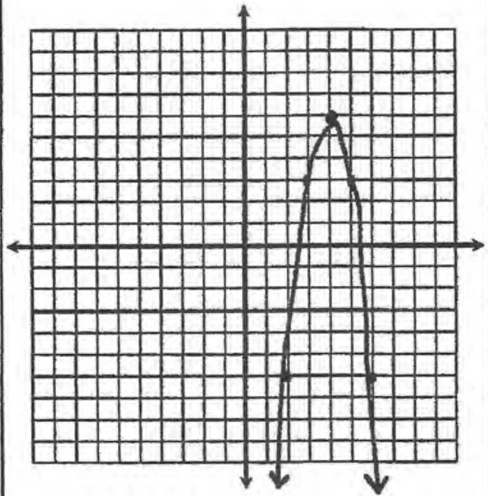
End Behavior:
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to \infty$

10

$f(x) = -3(x-4)^2 + 6$

Parent Function: f(x) = x²

Transformations: Vert. stretch by 3, Reflect in x-axis, Translate right 4 and up 6



Domain: ℝ	**Range:** {y | y ≤ 6}
x-intercept(s): (2.59, 0) (5.41, 0)	**y-intercept(s):** (0, -42)

Extrema: (4, 6) - Abs. Maximum

Increasing Interval(s): (-∞, 4)

Decreasing Interval(s): (4, ∞)

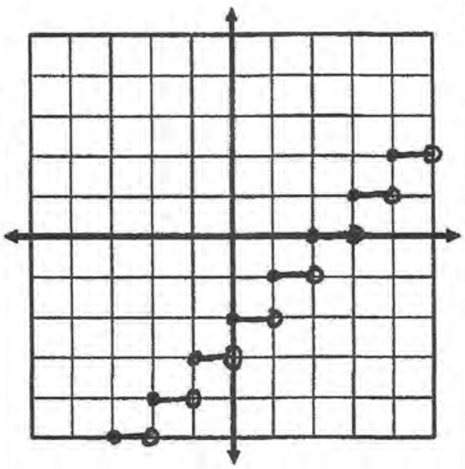
End Behavior:
As x → ∞, f(x) → -∞
As x → -∞, f(x) → -∞

11

$f(x) = [\![x+1]\!] - 3$

Parent Function: f(x) = ⟦x⟧

Transformations: Translate left 1, down 3



Domain: ℝ	**Range:** {y | y ∈ ℤ} (all integers)
x-intercept(s): (2, 0)	**y-intercept(s):** (0, -2)

Extrema: None

Increasing Interval(s): (-∞, ∞)

Decreasing Interval(s): None

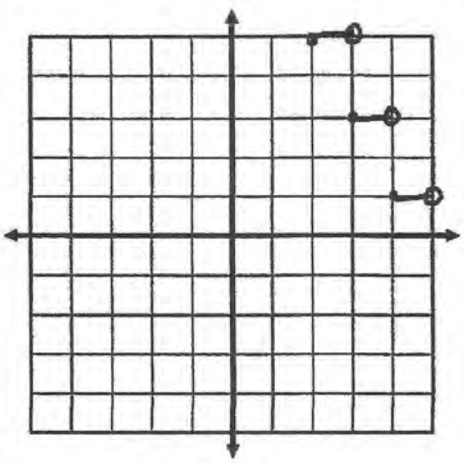
End Behavior:
As x → ∞, f(x) → ∞
As x → -∞, f(x) → -∞

12

$f(x) = -2[\![x-4]\!] + 1$

Parent Function: f(x) = ⟦x⟧

Transformations: Vertical stretch by 2, Reflect in x-axis, Translate right 4 and up 1



Domain: ℝ	**Range:** {y | y ∈ ℤ} (all integers)
x-intercept(s): None	**y-intercept(s):** (0, 9)

Extrema: None

Increasing Interval(s): None

Decreasing Interval(s): (-∞, ∞)

End Behavior:
As x → ∞ →, f(x) → -∞
As x → -∞, f(x) → ∞

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes	
Solving **RADICAL EQUATIONS**	①	**ISOLATE** the radical on one side of the equation.
	②	**RAISE EACH SIDE OF THE EQUATION TO THE POWER OF THE INDEX** to eliminate the radical sign.
	③	**SOLVE** the remaining equation.
	④	**CHECK** for extraneous solutions.

Directions: Solve each equation. Be sure to check for extraneous solutions.

1. $\sqrt{x}+5=12$

$-5 \quad -5$

$(\sqrt{x})^2=(7)^2$

$\boxed{x=49}$

Ck: $\sqrt{49}+5=12$

$7+5=12$ ✓

2. $3-\sqrt[4]{m}=0$

$+\sqrt[4]{m} \quad +\sqrt[4]{m}$

$(3)^4=(\sqrt[4]{m})^4$

$\boxed{81=m}$

Ck: $3-\sqrt[4]{81}=0$

$3-3=0$ ✓

3. $10+\sqrt{a+7}=11$

$-10 \quad -10$

$(\sqrt{a+7})^2=(1)^2$

$a+7=1$

$-7 \quad -7$

$\boxed{a=-6}$

Ck: $10+\sqrt{-6+7}=11$

$10+\sqrt{1}=11$

$10+1=11$ ✓

4. $\sqrt{6w-5}+10=3$

$-10 \quad -10$

$(\sqrt{6w-5})^2=(-7)^2$

$6w-5=49$

$+5 \quad +5$

$6w=54$

$\cancel{w=9}$

Ck: $\sqrt{6(9)-5}+10=3$

$\sqrt{54-5}+10=3$

$\sqrt{49}+10=3$

$7+10\neq 3$

$\boxed{\text{No Solution}}$

5. $-36=-9(c-12)^{\frac{1}{2}}$

$\frac{-36}{-9}=\frac{-9(c-12)^{\frac{1}{2}}}{-9}$

$(4)^2=\left((c-12)^{1/2}\right)^2$

$16=c-12$

$\boxed{28=c}$

Ck: $-36=-9(28-12)^{1/2}$

$-36=-9(16)^{1/2}$

$-36=-9(4)$ ✓

6. $(7p-1)^{\frac{1}{3}}+11=7$

$-11 \quad -11$

$\left((7p-1)^{1/3}\right)^3=(-4)^3$

$7p-1=-64$

$+1 \quad +1$

$7p=-63$

$\boxed{p=-9}$

Ck: $(7(-9)-1)^{1/3}+11=7$

$(-63-1)^{1/3}+11=7$

$(-64)^{1/3}+11=7$

$-4+11=7$ ✓

7. $\sqrt[4]{8k+12}-9=-11$

$+9 \quad +9$

$(\sqrt[4]{8k+12})^4=(-2)^4$

$8k+12=16$

$-12 \quad -12$

$8k=4$

$k=0.5$

Ck: $\sqrt[4]{8(0.5)+12}-9=-11$

$\sqrt[4]{4+12}-9=-11$

$\sqrt[4]{16}-9=-11$

$2-9\neq -11$

$\boxed{\text{No Solution}}$

8. $-4\sqrt{43-3x}+18=-2$

$-4\sqrt{43-3x}=-20$

$(\sqrt{43-3x})^2=(5)^2$

$43-3x=25$

$-3x=-18$

$\boxed{x=6}$

Ck: $-4\sqrt{43-3(6)}+18=-2$

$-4\sqrt{43-18}+18=-2$

$-4\sqrt{25}+18=-2$

$-4(5)+18=-2$

$-20+18=-2$ ✓

9. $\sqrt{u-8}=\sqrt{43-2u}$

$(\sqrt{u-8})^2=(\sqrt{43-2u})^2$

$u-8=43-2u$

$3u-8=43$

$3u=51$

$\boxed{u=17}$

Ck:

$\sqrt{17-8}=\sqrt{43-2(17)}$

$\sqrt{9}=\sqrt{43-34}$

$3=3$ ✓

10. $\sqrt[4]{27-2d}=\sqrt[4]{d-3}$

$(\sqrt[4]{27-2d})^4=(\sqrt[4]{d-3})^4$

$27-2d=d-3$

$27=3d-3$

$30=3d$

$\boxed{10=d}$

Ck:

$\sqrt[4]{27-2(10)}=\sqrt[4]{10-3}$

$\sqrt[4]{27-20}=\sqrt[4]{7}$

$\sqrt[4]{7}=\sqrt[4]{7}$ ✓

11. $\frac{4\sqrt{3q+15}}{4}=\frac{12\sqrt{q}}{4}$

$(\sqrt{3q+15})^2=(3\sqrt{q})^2$

$3q+15=9q$

$-3q \quad -3q$

$15=6q$

$\boxed{\frac{5}{2}=q}$

Ck:

$4\sqrt{3(\frac{5}{2})+15}=12\sqrt{\frac{5}{2}}$

$4\sqrt{\frac{15}{2}+15}=12\sqrt{\frac{5}{2}}$

$4\sqrt{\frac{45}{2}}=12\sqrt{\frac{5}{2}}$

$4\frac{\sqrt{9}\sqrt{5}}{\sqrt{2}}=12\frac{\sqrt{5}}{\sqrt{2}}$

$12\sqrt{\frac{5}{2}}=12\sqrt{\frac{5}{2}}$ ✓

12. $(\sqrt[3]{14-10n})^3=(-2\sqrt[3]{n})^3$

$14-10n=-8n$

$14=2n$

$\boxed{7=n}$

Ck:

$\sqrt[3]{14-10(7)}=-2\sqrt[3]{7}$

$\sqrt[3]{-56}=-2\sqrt[3]{7}$

$\sqrt[3]{-8}\sqrt[3]{7}=-2\sqrt[3]{7}$

$-2\sqrt[3]{7}=-2\sqrt[3]{7}$ ✓

13. $(\sqrt{8x})^2=(x)^2$

$8x=x^2$

$0=x^2-8x$

$0=x(x-8)$

$\boxed{x=0}$ | $\boxed{x=8}$

Ck: $\sqrt{8\cdot 0}=\sqrt{0}$

$0=0$ ✓

$\sqrt{8\cdot 8}=8$

$\sqrt{64}=8$ ✓

14. $\sqrt{12-a}+7a=8a$

$(\sqrt{12-a})^2=(a)^2$

$12-a=a^2$

$a^2+a-12=0$

$(a+4)(a-3)=0$

$\cancel{a=-4}$ | $\boxed{a=3}$

Ck: $\sqrt{12-(-4)}+7(-4)=8(-4)$

$\sqrt{16}-28=-32$

$4-28\neq-32$

$\sqrt{12-3}+7(3)=8(3)$

$\sqrt{9}+21=24$

$3+21=24$ ✓

15. $(\sqrt{6m-38})^2=(m-5)^2$

$6m-38=m^2-10m+25$

$0=m^2-16m+63$

$0=(m-9)(m-7)$

$\boxed{m=9}$ | $\boxed{m=7}$

Ck:

$\sqrt{6(9)-38}=9-5$

$\sqrt{54-38}=4$

$\sqrt{16}=4$ ✓

$\sqrt{6(7)-38}=7-5$

$\sqrt{42-38}=2$

$\sqrt{4}=2$ ✓

16. $(\sqrt{2v+3})^2=(v+2)^2$

$2v+3=v^2+4v+4$

$0=v^2+2v+1$

$0=(v+1)(v+1)$

$\boxed{v=-1}$ | $\boxed{v=-1}$

Ck:

$\sqrt{2(-1)+3}=-1+2$

$\sqrt{-2+3}=1$

$\sqrt{1}=1$ ✓

17. $(\sqrt{51-5y})^2=(y-11)^2$

$51-5y=y^2-22y+121$

$0=y^2-17y+70$

$0=(y-10)(y-7)$

$\cancel{y=10}$ | $y\neq7$

$\boxed{\text{No Solution}}$

Ck:

$\sqrt{51-5(10)}=10-11$

$\sqrt{51-50}=-1$

$\sqrt{1}\neq-1$

$\sqrt{51-5(7)}=7-11$

$\sqrt{51-35}=-4$

$\sqrt{16}\neq-4$

18. $(\sqrt{x+9})^2=(\sqrt{x}-1)^2$

$x+9=x-2\sqrt{x}+1$

$9=-2\sqrt{x}+1$

$8=-2\sqrt{x}$

$(-4)^2=(\sqrt{x})^2$

$\cancel{16=x}$

$\boxed{\text{No Solution}}$

Ck:

$\sqrt{16+9}=\sqrt{16}-1$

$\sqrt{25}=4-1$

$5\neq3$

REVIEW: Radicals & Rational Exponents

SIMPLIFYING RADICALS

Write each expression in simplest radical form.

1. $\sqrt{300m^4n^{25}}$

$\sqrt{100m^4n^{24}}\ \sqrt{3n}$

$=\boxed{10m^2n^{12}\sqrt{3n}}$

2. $\sqrt[3]{-72a^9b^2}$

$\sqrt[3]{-8a^9}\ \sqrt[3]{9b^2}$

$=\boxed{-2a^3\sqrt[3]{9b^2}}$

3. $\sqrt[4]{81m^{11}n^{20}}$

$\sqrt[4]{81m^8n^{20}}\ \sqrt[4]{m^3}$

$=\boxed{3m^2n^5\sqrt[4]{m^3}}$

OPERATIONS WITH RADICALS

Perform the indicated operation(s). Write each answer in simplest radical form.

4. $4\sqrt{98}+\sqrt{150}-2\sqrt{32}$

$4\sqrt{49}\sqrt{2}+\sqrt{25}\sqrt{6}-2\sqrt{16}\sqrt{2}$

$=28\sqrt{2}+5\sqrt{6}-8\sqrt{2}$

$=\boxed{20\sqrt{2}+5\sqrt{6}}$

5. $10\sqrt[4]{5}+7\sqrt[3]{40}-\sqrt[3]{320}$

$10\sqrt[4]{5}+7\sqrt[3]{8}\sqrt[3]{5}-\sqrt[3]{64}\sqrt[3]{5}$

$=10\sqrt[4]{5}+14\sqrt[3]{5}-4\sqrt[3]{5}$

$=\boxed{10\sqrt[4]{5}+10\sqrt[3]{5}}$

6. $3\sqrt[4]{8}\cdot 7\sqrt[4]{32}$

$21\sqrt[4]{256}$

$=21\cdot 4 \quad =\boxed{84}$

7. $\sqrt[3]{-6x^5}\cdot\sqrt[3]{18x^4}$

$\sqrt[3]{-108x^9} \quad =\sqrt[3]{-27x^9}\ \sqrt[3]{4}$

$=\boxed{-3x^3\sqrt[3]{4}}$

8. $\sqrt{3}(2-\sqrt{24})$

$2\sqrt{3}-\sqrt{72}$

$=2\sqrt{3}-\sqrt{36}\sqrt{2}$

$=\boxed{2\sqrt{3}-6\sqrt{2}}$

9. $(4+3\sqrt{5})\cdot(-2+\sqrt{5})$

$-8+4\sqrt{5}-6\sqrt{5}+3\sqrt{25}$

$=-8-2\sqrt{5}+15$

$=\boxed{7-2\sqrt{5}}$

10. $(1+7\sqrt{3})(1-7\sqrt{3})$

$1-7\sqrt{3}+7\sqrt{3}-49\sqrt{9}$

$1-147$

$=\boxed{-146}$

11. $(3+\sqrt{6})^2$

$(3+\sqrt{6})(3+\sqrt{6})$

$=9+3\sqrt{6}+3\sqrt{6}+\sqrt{36}$

$=9+6\sqrt{6}+6 \quad =\boxed{15+6\sqrt{6}}$

12. $\dfrac{-45\sqrt{156}}{9\sqrt{3}}$

$=-5\sqrt{52}$

$=-5\sqrt{4}\sqrt{13}$

$=\boxed{-10\sqrt{13}}$

13. $\dfrac{\sqrt[4]{r^{20}s^9}}{\sqrt[4]{r^3s}}=\sqrt[4]{r^{17}s^8}$

$=\sqrt[4]{r^{16}s^8}\ \sqrt[4]{r}$

$=\boxed{r^4s^2\sqrt[4]{r}}$

14. $\sqrt{\frac{50}{81}} = \frac{\sqrt{25}\sqrt{2}}{\sqrt{81}} = \boxed{\frac{5\sqrt{2}}{9}}$	15. $\sqrt[4]{\frac{3a}{16b^8}}$ $\frac{\sqrt[4]{3a}}{\sqrt[4]{16b^8}} = \boxed{\frac{\sqrt[4]{3a}}{2b^2}}$
16. $\frac{\sqrt{6}}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{18}}{4\sqrt{9}} = \frac{\sqrt{9}\sqrt{2}}{4\sqrt{9}} = \boxed{\frac{\sqrt{2}}{4}}$	17. $\sqrt{\frac{8x}{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{40x}}{\sqrt{25}} = \frac{\sqrt{4}\sqrt{10x}}{5} = \boxed{\frac{2\sqrt{10x}}{5}}$
18. $\frac{(2-\sqrt{12})\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{36}}{\sqrt{9}} = \boxed{\frac{2\sqrt{3}-6}{3}}$	19. $\frac{3+\sqrt{3}}{4\sqrt{12}} = \frac{3+\sqrt{3}}{4\sqrt{4}\sqrt{3}} = \frac{(3+\sqrt{3})\cdot\sqrt{3}}{(8\sqrt{3})\cdot\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{9}}{8\sqrt{9}} = \boxed{\frac{\sqrt{3}+1}{8}}$
20. $\frac{(5-4\sqrt{2})(4+3\sqrt{2})}{(4-3\sqrt{2})(4+3\sqrt{2})} = \frac{20+15\sqrt{2}-16\sqrt{2}-12\sqrt{4}}{16+12\sqrt{2}-12\sqrt{2}-9\sqrt{4}} = \frac{20-\sqrt{2}-24}{16-18} = \frac{-4-\sqrt{2}}{-2} = \boxed{\frac{4+\sqrt{2}}{2}}$	21. $\frac{(2+3\sqrt{3})(\sqrt{3}-5)}{(\sqrt{3}+5)(\sqrt{3}-5)} = \frac{2\sqrt{3}-10+3\sqrt{9}-15\sqrt{3}}{\sqrt{9}-5\sqrt{3}+5\sqrt{3}-25} = \frac{-13\sqrt{3}-10+9}{-22} = \frac{-1-13\sqrt{3}}{-22} = \boxed{\frac{1+13\sqrt{3}}{22}}$

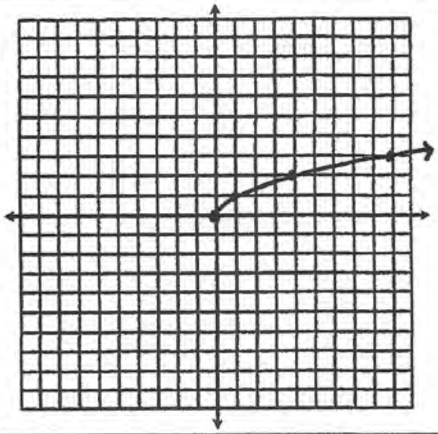
RATIONAL EXPONENTS

Simplify each expression. Write all final answers in simplest radical form.

22. $x^{\frac{2}{3}} \cdot x^{\frac{11}{3}} = x^{13/3} = \sqrt[3]{x^{13}} = \sqrt[3]{x^{12}} \cdot \sqrt[3]{x} = \boxed{x^4\sqrt[3]{x}}$	23. $32^{\frac{1}{8}} \cdot 32^{\frac{3}{8}} = 32^{4/8} = 32^{1/2} = \sqrt{32} = \sqrt{16}\sqrt{2} = \boxed{4\sqrt{2}}$	24. $n^{\frac{7}{3}} \cdot n^3 = n^{\frac{16}{3}} = \sqrt[3]{n^{16}} = \sqrt[3]{n^{15}}\sqrt[3]{n} = \boxed{n^5\sqrt[3]{n}}$
25. $\frac{7^{\frac{9}{4}}}{7^{\frac{1}{4}}} = 7^{8/4} = 7^2$ $\boxed{=49}$	26. $\left(k^{\frac{3}{2}}\right)^{\frac{1}{2}} = k^{3/4} = \boxed{\sqrt[4]{k^3}}$	27. $1{,}296^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{1296}} = \boxed{\frac{1}{6}}$
28. $\frac{125^{\frac{1}{3}}}{125^{\frac{2}{3}}} = \frac{1}{125^{1/3}} = \frac{1}{\sqrt[3]{125}} = \boxed{\frac{1}{5}}$	29. $\frac{\sqrt[4]{3^3}}{\sqrt{3}} = \frac{3^{3/4}}{3^{1/2}} = 3^{1/4} = \boxed{\sqrt[4]{3}}$	30. $\sqrt[4]{10^2 a^{18}}$ $(10^2a^{18})^{1/4} = 10^{1/2}a^{9/2} = \boxed{a^4\sqrt{10a}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
SQUARE ROOT Function	A **radical function** is a function of the form $f(x)=\sqrt[n]{x}$. The **square root function** is a type of radical function.
Parent Function: $\boxed{f(x)=\sqrt{x}}$	**Graph the parent function of the square root function below and identify the key characteristics.**



D: $\{x \mid x \geq 0\}$ R: $\{y \mid y \geq 0\}$

Endpoint: $(0,0)$

End Behavior:

As $x \to$ ∞, $f(x) \to$ ∞

As $x \to$ 0, $f(x) \to$ 0

Increasing Interval(s): $[0, \infty)$

Decreasing Interval(s): N/A

TRANFORMATIONS

$\boxed{f(x)=a\sqrt{x-h}+k}$

- h is the horizontal shift. (+ shifts left, – shifts right)
- k is the vertical shift. (+ shifts up, – shifts down)
- Endpoint: (h, k)
- If a is negative, the function is reflected across the x-axis
- $|a|>1$ represents a vertical stretch.
- $0<|a|<1$ represents a vertical compression

Describe the transformations of each function compared to the parent function.

1. $f(x)=\sqrt{x-7}+2$ Right 7, up 2	**2.** $f(x)=4\sqrt{x}-1$ Vert. stretch by 4, down 1
3. $f(x)=-\sqrt{x+3}$ Reflect across x-axis, left 3	**4.** $f(x)=\frac{1}{2}\sqrt{x-2}-6$ Vert. compression by $\frac{1}{2}$, right 2, down 6

5. The square root parent function is reflected across the x-axis, then shifted 7 units left and 1 unit down. Write an equation that represents this function.

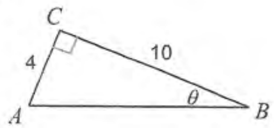
$f(x) = -\sqrt{x+7} - 1$

Precalculus Summer Packet Name________________________

Right Triangles - Week 6 (2 problems/day)

Find the measure of each angle indicated. Round to the nearest tenth.

1)

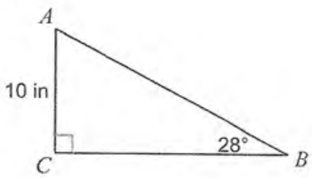


In each problem, angle C is a right angle. Find the angle indicated to the nearest tenth.

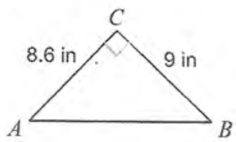
2) Find $m\angle B$ if $a = 14,\ b = 10$

Solve each triangle. Round answers to the nearest tenth.

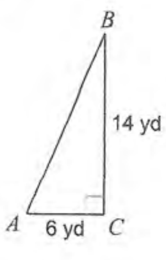
3)



4)



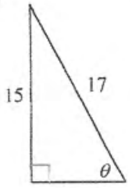
5)



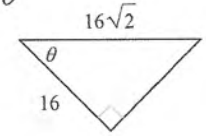
Week 6 - (con't)

Find the value of the trig function indicated.

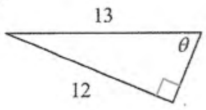
6) $\sin\theta$



7) $\sin\theta$



8) $\cos\theta$



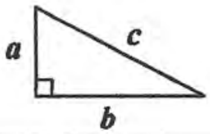
Name:

Date:

Topic:

Class:

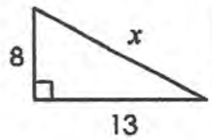
Main Ideas/Questions	Notes/Examples
PYTHAGOREAN THEOREM	• Used to find the missing side of a right triangle. • Sides $\underline{a}$ and $\underline{b}$ are called legs. • Side $\underline{c}$ is called the hypotenuse. • For any right triangle: $a^2+b^2=c^2$.



Examples

Directions: Find the value of x. Round your answer to the nearest tenth.

1.



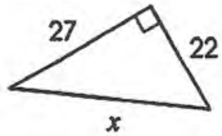
$8^2+13^2=x^2$

$64+169=x^2$

$233=x^2$

$\boxed{x=15.3}$ ($\sqrt{233}$)

2.



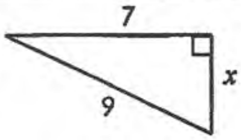
$22^2+27^2=x^2$

$484+729=x^2$

$1213=x^2$

$\boxed{x=34.8}$ ($\sqrt{1213}$)

3.



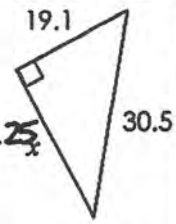
$7^2+x^2=9^2$

$49+x^2=81$

$x^2=32$

$\boxed{x=5.7}$ ($4\sqrt{2}$)

4.



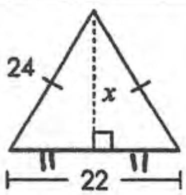
$19.1^2+x^2=30.5^2$

$364.81+x^2=930.25$

$x^2=565.44$

$\boxed{x=23.8}$

5.



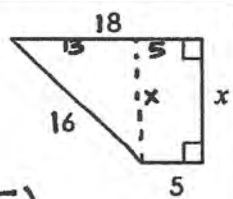
$11^2+x^2=24^2$

$121+x^2=576$

$x^2=455$

$\boxed{x=21.3}$ ($\sqrt{455}$)

6.



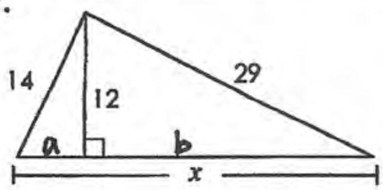
$x^2+13^2=16^2$

$x^2+169=256$

$x^2=87$

$\boxed{x=9.3}$ ($\sqrt{87}$)

7.



$12^2+a^2=14^2$

$144+a^2=196$

$a^2=52$

$a=7.2$

$12^2+b^2=29^2$

$144+b^2=841$

$b^2=697$

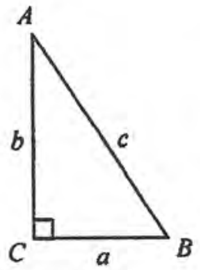
$b=26.4$

$x=7.2+26.4=\boxed{33.6}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
What is **TRIGONOMETRY?**	The study of triangle measurement.

TRIGONOMETRIC RATIOS



Each acute angle of a right triangle has the following trigonometric ratios:

SINE	The ratio of the leg **opposite** the angle to the **hypotenuse.**	• $\sin A = a/c$ • $\sin B = b/c$
COSINE	The ratio of the leg **adjacent** to the angle to the **hypotenuse.**	• $\cos A = b/c$ • $\cos B = a/c$
TANGENT	The ratio of the leg **opposite** the angle to the leg **adjacent** to the angle.	• $\tan A = a/b$ • $\tan B = b/a$

* REMEMBER!! *

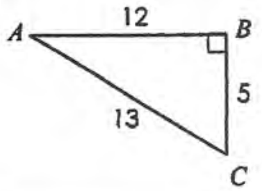
SOH CAH TOA

$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj}$

EXAMPLES

Directions: Give each trigonometric ratio as a fraction in simplest form.

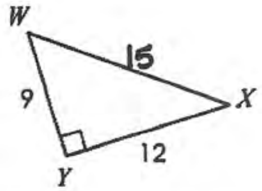
1.



- $\sin A = 5/13$
- $\cos A = 12/13$
- $\tan A = 5/12$
- $\sin C = 12/13$
- $\cos C = 5/13$
- $\tan C = 12/5$

$9^2+12^2=c^2$
$225=c^2$
$15=c$

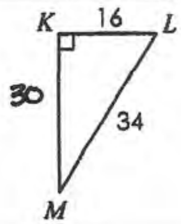
2.



- $\sin W = 12/15 = 4/5$
- $\cos W = 9/15 = 3/5$
- $\tan W = 12/9 = 4/3$
- $\sin X = 9/15 = 3/5$
- $\cos X = 12/15 = 4/5$
- $\tan X = 9/12 = 3/4$

$16^2+b^2=34^2$
$b^2=900$
$b=30$

3.



- $\sin L = 30/34 = 15/17$
- $\cos L = 16/34 = 8/17$
- $\tan L = 30/16 = 15/8$
- $\sin M = 16/34 = 8/17$
- $\cos M = 30/34 = 15/17$
- $\tan M = 16/30 = 8/15$

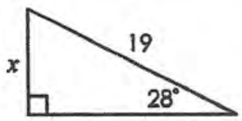
FINDING SIDE LENGTHS

using Trig

Note: Make sure your calculator is in **degree mode**!

Directions: Solve for x. Round to the nearest tenth.

4.

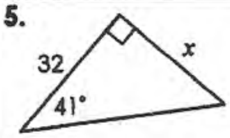


$\sin 28 = \frac{x}{19}$

$x = 19 \cdot \sin 28$

$\boxed{x = 8.9}$

5.

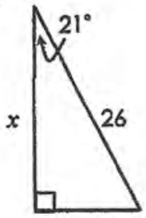


$\tan 41 = \frac{x}{32}$

$32 \cdot \tan 41 = x$

$\boxed{x = 27.8}$

6.

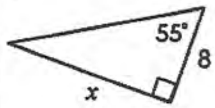


$\cos 21 = \frac{x}{26}$

$x = 26 \cdot \cos 21$

$\boxed{x = 24.3}$

7.

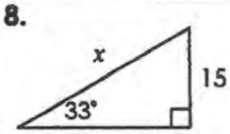


$\tan 55 = \frac{x}{8}$

$x = 8 \cdot \tan 55$

$\boxed{x = 11.4}$

8.

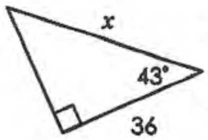


$\sin 33 = \frac{15}{x}$

$\frac{x \sin 33}{\sin 33} = \frac{15}{\sin 33}$

$\boxed{x = 27.5}$

9.

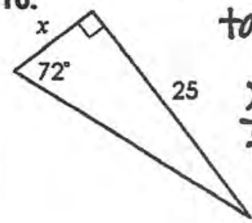


$\cos 43 = \frac{36}{x}$

$\frac{x \cos 43}{\cos 43} = \frac{36}{\cos 43}$

$\boxed{x = 49.2}$

10.

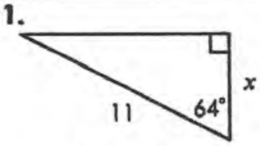


$\tan 72 = \frac{25}{x}$

$\frac{x \tan 72}{\tan 72} = \frac{25}{\tan 72}$

$\boxed{x = 8.1}$

11.

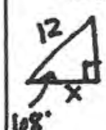


$\cos 64 = \frac{x}{11}$

$x = 11 \cos 64$

$\boxed{x = 4.8}$

12. Jake leaned a 12-foot ladder against his house. If the angle formed by the ladder and the ground is 68°, how far from the base of the house did he place the ladder?

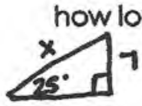


$\cos 68 = \frac{x}{12}$

$12 \cos 68 = x$

$\boxed{x = 4.5 \text{ ft}}$

13. A ramp is used to load suitcases on an airplane. If the cargo door is 7 feet from the ground and the angle formed by the end of the ramp and the ground is 25°, how long is the ramp?



$\sin 25 = \frac{7}{x}$

$\frac{x \sin 25}{\sin 25} = \frac{7}{\sin 25}$

$\boxed{x = 16.6 \text{ ft}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions | **Notes/Examples**

FINDING ANGLE MEASURES

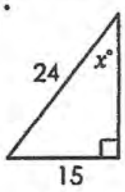
using Trig

Note: Make sure your calculator is in **degree mode!**

If you know the sin, cosine, or tangent ratio of an angle, you can use the inverse of the ratio ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$) to find the measure of the angle.

Directions: Find the value of x. Round to the nearest tenth.

1.

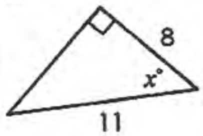


$\sin x = \frac{15}{24}$

$x = \sin^{-1}\left(\frac{15}{24}\right)$

$\boxed{x = 38.7°}$

2.

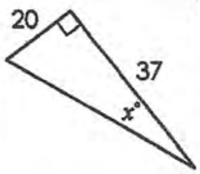


$\cos x = \frac{8}{11}$

$x = \cos^{-1}\left(\frac{8}{11}\right)$

$\boxed{x = 433°}$

3.

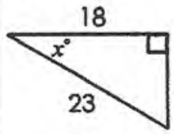


$\tan x = \frac{20}{37}$

$x = \tan^{-1}\left(\frac{20}{37}\right)$

$\boxed{x = 28.4°}$

4.

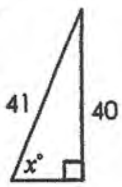


$\cos x = \frac{18}{23}$

$x = \cos^{-1}\left(\frac{18}{23}\right)$

$\boxed{x = 38.5°}$

5.

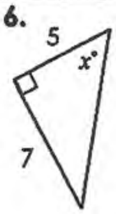


$\sin x = \frac{40}{41}$

$x = \sin^{-1}\left(\frac{40}{41}\right)$

$\boxed{x = 77.3°}$

6.



$\tan x = \frac{7}{5}$

$x = \tan^{-1}\left(\frac{7}{5}\right)$

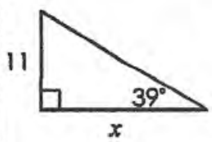
$\boxed{x = 54.5°}$

REVIEW:

Sides & Angles

Directions: Find the value of x. Round to the nearest tenth.

7.

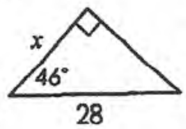


$\tan 39 = \frac{11}{x}$

$x = \frac{11}{\tan 39}$

$\boxed{x = 13.6}$

8.

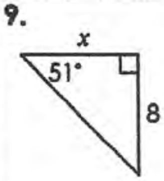


$\cos 46 = \frac{x}{28}$

$28 \cos 46 = x$

$\boxed{x = 19.5}$

9.

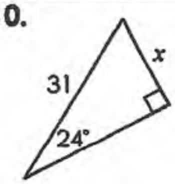


$\tan 51 = \frac{8}{x}$

$x = \frac{8}{\tan 51}$

$\boxed{x = 6.5}$

10.



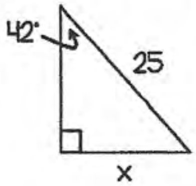
$\sin 24 = \frac{x}{31}$

$31 \sin 24 = x$

$\boxed{x = 12.6}$

Finding missing SIDES with TRIGONOMETRY

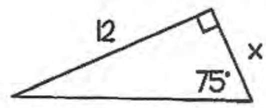
EXAMPLE 1



$\sin 42 = \frac{x}{25}$

$\boxed{x = 16.7}$

EXAMPLE 2

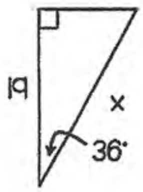


$\tan 75 = \frac{12}{x}$

$x = \frac{12}{\tan 75}$

$\boxed{x = 3.2}$

EXAMPLE 3

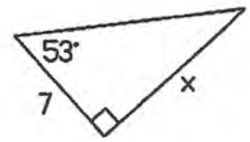


$\cos 36 = \frac{19}{x}$

$x = \frac{19}{\cos 36}$

$\boxed{x = 23.5}$

EXAMPLE 4

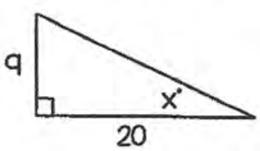


$\tan 53 = \frac{x}{7}$

$\boxed{x = 9.3}$

Finding missing ANGLES with TRIGONOMETRY

EXAMPLE 1

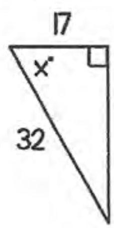


$\tan x = \frac{9}{20}$

$x = \tan^{-1}\left(\frac{9}{20}\right)$

$\boxed{x = 24.2^\circ}$

EXAMPLE 2

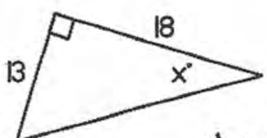


$\cos x = \frac{17}{32}$

$x = \cos^{-1}\left(\frac{17}{32}\right)$

$\boxed{x = 57.9^\circ}$

EXAMPLE 3

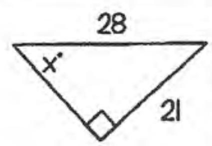


$\tan x = \frac{13}{18}$

$x = \tan^{-1}\left(\frac{13}{18}\right)$

$\boxed{x = 35.8^\circ}$

EXAMPLE 4



$\sin x = \frac{21}{28}$

$x = \sin^{-1}\left(\frac{21}{28}\right)$

$\boxed{x = 48.6^\circ}$

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Week 7 - Operations with Radicals (3-4 problems/day)

Simplify.

1) $\sqrt{216}$

2) $\sqrt{256}$

3) $\sqrt{8m^2}$

4) $\sqrt{80a^3}$

5) $\sqrt{18a^4b^4}$

6) $\sqrt{512x^3y^3}$

7) $\sqrt[3]{750x^6y^8}$

8) $\sqrt[3]{16x^8y^3}$

9) $-\sqrt{54}-\sqrt{54}$

10) $-2\sqrt{8}-3\sqrt{2}$

11) $-3\sqrt[3]{32}-2\sqrt[3]{108}$

12) $3\sqrt[4]{2}+2\sqrt[4]{2}$

13) $\sqrt{6}\cdot\sqrt{10}$

14) $-5\sqrt{5}\cdot\sqrt{25}$

15) $-2\sqrt{15}\left(4\sqrt{5}+5\sqrt{6}\right)$

16) $\sqrt{15}\left(\sqrt{2}+\sqrt{5}\right)$

17) $\dfrac{\sqrt{12}}{\sqrt{25}}$

18) $\dfrac{2\sqrt{12}}{\sqrt{25}}$

19) $\dfrac{\sqrt{4}}{\sqrt{5}}$

20) $\dfrac{3\sqrt{2}}{5\sqrt{3}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

WARM-UP

List the perfect squares, cubes, and fourths.

Perfect Squares:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Perfect Cubes:
1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, ...

Perfect Fourths:
1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10000, ...

NTH ROOTS

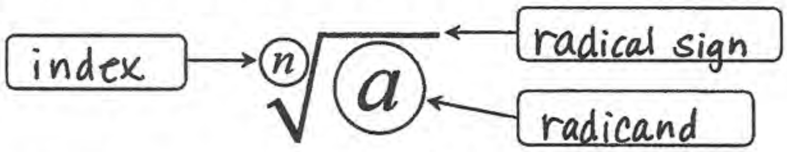
Definition: x is the n^{th} **root** of a real number a if $x^n = a$

Examples:

- 9 and -9 are **square roots** of 81 because $9^2 = 81$ and $(-9)^2 = 81$
- -2 is the **cube root** of -8 because $(-2)^3 = -8$
- 4 and -4 are **fourth roots** of 256 because $4^4 = 256$ and $(-4)^4 = 256$

RADICAL Expression

The n^{th} root of a real number, a, can be written as the radical expression $\sqrt[n]{a}$



- If there is **no index**, it is assumed that $n = 2$.
- **Number of Roots:**

Index	Radicand	Type of Roots	# of Roots
Even	Positive	real	2 (±)
Odd	Positive	real	1 (+)
Odd	Negative	real	1 (-)
★ Even	Negative	imaginary	2 (±)

- If a radicand has more than one n^{th} root, the radical sign indicates only the **principal, or positive, root.**

EVALUATING Radicals

Find each value.

$\sqrt{16} = 4$	$-\sqrt{121} = -11$	$\sqrt{289} = 17$	$-\sqrt{\frac{4}{25}} = -\frac{2}{5}$
$\sqrt[3]{8} = 2$	$\sqrt[3]{343} = 7$	$\sqrt[3]{-125} = -5$	$\sqrt[3]{-\frac{1}{27}} = -\frac{1}{3}$
$-\sqrt[4]{1} = -1$	$\sqrt[4]{2{,}401} = 7$	$-\sqrt[4]{4{,}096} = -8$	$\sqrt[4]{\frac{81}{16}} = \frac{3}{2}$

SIMPLIFYING Radicals

1. $\sqrt{117}$ $= \sqrt{9}\cdot\sqrt{13}$

$\boxed{= 3\sqrt{13}}$

2. $4\sqrt{320}$ $= 4\cdot\sqrt{64}\cdot\sqrt{5}$

$= 4\cdot 8\cdot\sqrt{5}$

$\boxed{= 32\sqrt{5}}$

3. $2\sqrt[3]{48}$ $= 2\cdot\sqrt[3]{8}\cdot\sqrt[3]{6}$

$= 2\cdot 2\cdot\sqrt[3]{6}$

$\boxed{= 4\sqrt[3]{6}}$

4. $3\sqrt[3]{108}$ $= 3\cdot\sqrt[3]{27}\cdot\sqrt[3]{4}$

$= 3\cdot 3\cdot\sqrt[3]{4}$

$\boxed{= 9\sqrt[3]{4}}$

5. $\sqrt[3]{-250}$ $= \sqrt[3]{-125}\cdot\sqrt[3]{2}$

$\boxed{= -5\sqrt[3]{2}}$

6. $6\sqrt[3]{-2}$ $= 6\cdot\sqrt[3]{-1}\cdot\sqrt[3]{2}$

$= 6\cdot(-1)\cdot\sqrt[3]{2}$

$\boxed{= -6\sqrt[3]{2}}$

7. $3\sqrt[4]{162}$ $= 3\cdot\sqrt[4]{81}\cdot\sqrt[4]{2}$

$= 3\cdot 3\cdot\sqrt[4]{2}$

$\boxed{= 9\sqrt[4]{2}}$

8. $5\sqrt[4]{2,592}$ $= 5\cdot\sqrt[4]{1296}\cdot\sqrt[4]{2}$

$= 5\cdot 6\cdot\sqrt[4]{2}$

$\boxed{= 30\sqrt[4]{2}}$

Radicals with VARIABLES

Square Roots	Cube Roots	4th Roots
Exponents must be multiples of 2!	Exponents must be multiples of 3!	Exponents must be multiples of 4!

9. $\sqrt{32x^4y^9}$ $= \sqrt{16x^4y^8}\cdot\sqrt{2y}$

$\boxed{= 4x^2y^4\sqrt{2y}}$

10. $\sqrt{324a^3b^7}$ $= \sqrt{324a^2b^6}\cdot\sqrt{ab}$

$\boxed{= 18ab^3\sqrt{ab}}$

11. $\sqrt[3]{216m^3n^6}$ $\boxed{= 6mn^2}$

12. $\sqrt[3]{56r^8s^4}$ $= \sqrt[3]{8r^6s^3}\cdot\sqrt[3]{7r^2s}$

$\boxed{= 2r^2s\sqrt[3]{7r^2s}}$

13. $\sqrt[3]{-64x^{10}y^{21}}$ $= \sqrt[3]{-64x^9y^{21}}\cdot\sqrt[3]{x}$

$\boxed{= -4x^3y^7\sqrt[3]{x}}$

14. $\sqrt[3]{-81p^2q^{12}}$ $= \sqrt[3]{-27q^{12}}\cdot\sqrt[3]{3p^2}$

$\boxed{= -3q^4\sqrt[3]{3p^2}}$

15. $\sqrt[4]{w^4v^{17}}$ $= \sqrt[4]{w^4v^{16}}\cdot\sqrt[4]{v}$

$\boxed{= wv^4\sqrt[4]{v}}$

16. $\sqrt[4]{48m^8n^3}$ $= \sqrt[4]{16m^8}\cdot\sqrt[4]{3n^3}$

$\boxed{= 2m^2\sqrt[4]{3n^3}}$

Name:	Date:
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Main Ideas/Questions	Notes/Examples		
RATIONAL EXPONENTS	Expressions with rational exponents can be rewritten as radicals using the following rules:		
	Exponential Form	**Meaning**	**Radical Form**
	$a^{\frac{1}{n}}$	The n^{th} root of a	$a^{\frac{1}{n}} = \sqrt[n]{a}$
	$a^{\frac{m}{n}}$	The n^{th} root of a, raised to the m^{th} power	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Converting EXPONENTIAL TO RADICAL FORM

Directions: Write each expression in **radical form**. Simplify if needed.

1. $x^{\frac{1}{4}}$ $\boxed{= \sqrt[4]{x}}$	**2.** $24^{\frac{1}{3}}$ $= \sqrt[3]{24}$ $= \sqrt[3]{8} \cdot \sqrt[3]{3}$ $\boxed{= 2\sqrt[3]{3}}$	**3.** $(15x)^{\frac{1}{2}}$ $\boxed{= \sqrt{15x}}$
4. $7^{\frac{2}{3}}$ $= \sqrt[3]{7^2}$ $\boxed{= \sqrt[3]{49}}$	**5.** $k^{\frac{7}{2}}$ $= \sqrt{k^7}$ $= \sqrt{k^6} \cdot \sqrt{k}$ $\boxed{= k^3\sqrt{k}}$	**6.** $3^{\frac{5}{4}}$ $= \sqrt[4]{3^5}$ $= \sqrt[4]{3^4} \cdot \sqrt[4]{3}$ $\boxed{= 3\sqrt[4]{3}}$
7. $(ab)^{\frac{3}{4}}$ $= \sqrt[4]{(ab)^3}$ $\boxed{= \sqrt[4]{a^3b^3}}$	**8.** $(-6x)^{\frac{2}{3}}$ $\sqrt[3]{(-6x)^2}$ $= \boxed{\sqrt[3]{36x^2}}$	**9.** $7(12w)^{\frac{1}{2}}$ $= 7\sqrt{12w}$ $= 7\sqrt{4}\sqrt{3w}$ $= \boxed{14\sqrt{3w}}$

Converting RADICAL TO EXPONENTIAL FORM

Directions: Write each expression in **exponential form.**

10. $\sqrt[3]{16}$ $\boxed{= 16^{1/3}}$	**11.** $\sqrt{xy}$ $= (xy)^{1/2}$ $\boxed{= x^{1/2}y^{1/2}}$	**12.** $\sqrt[4]{8w^2}$ $= (8w^2)^{1/4}$ $\boxed{= 8^{1/4}w^{1/2}}$
13. $\sqrt[3]{11^2}$ $\boxed{= 11^{2/3}}$	**14.** $\sqrt[4]{k^{10}}$ $= k^{10/4}$ $\boxed{= k^{5/2}}$	**15.** $(\sqrt{3m})^7$ $= (3m)^{7/2}$ $= \boxed{3^{\frac{7}{2}}m^{\frac{7}{2}}}$
16. $(\sqrt[4]{-2a})^5$ $= (-2a)^{5/4}$ $\boxed{= (-2)^{5/4} \cdot a^{5/4}}$	**17.** $\sqrt{10^5a^3b^8}$ $= (10^5a^3b^8)^{1/2}$ $\boxed{= 10^{\frac{5}{2}}a^{\frac{3}{2}}b^4}$	**18.** $\sqrt[8]{9x^2y^{12}}$ $= (9x^2y^{12})^{1/8}$ $\boxed{= 9^{1/8}x^{1/4}y^{3/2}}$

SIMPLIFYING EXPRESSIONS

with Rational Exponents

①	Rewrite all radicals in **exponential form.**
②	Use the **exponent rules** to simplify the expression.
③	Write your **answer as a radical in simplest form.** Simplify if needed.

19. $x^{\frac{1}{3}} \cdot x^{\frac{4}{3}} = x^{5/3}$

$= \sqrt[3]{x^5}$

$= \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = \boxed{x\sqrt[3]{x^2}}$

20. $p^{\frac{1}{4}} \cdot p^{\frac{3}{2}} = p^{\frac{1}{4}} \cdot p^{\frac{6}{4}} = p^{\frac{7}{4}}$

$= \sqrt[4]{p^7}$

$= \sqrt[4]{p^4} \cdot \sqrt[4]{p^3} = \boxed{p\sqrt[4]{p^3}}$

21. $\dfrac{m^{\frac{5}{2}}}{m^{\frac{7}{4}}} = \dfrac{m^{\frac{10}{4}}}{m^{\frac{7}{4}}} = m^{\frac{3}{4}}$

$= \boxed{\sqrt[4]{m^3}}$

22. $\left(a^{\frac{1}{3}}\right)^{\frac{5}{2}} = a^{5/6}$

$= \boxed{\sqrt[6]{a^5}}$

23. $\left(32^{\frac{1}{2}}\right)^{\frac{1}{2}} = 32^{1/4}$

$= \sqrt[4]{32}$

$= \sqrt[4]{16} \cdot \sqrt[4]{2} = \boxed{2\sqrt[4]{2}}$

24. $(8x^2)^{\frac{2}{3}} = 8^{2/3}x^{4/3}$

$= \sqrt[3]{64x^4}$

$= \sqrt[3]{64x^3} \cdot \sqrt[3]{x} = \boxed{4x\sqrt[3]{x}}$

25. $100^{-\frac{1}{2}} = \dfrac{1}{100^{1/2}}$

$= \dfrac{1}{\sqrt{100}} = \boxed{\dfrac{1}{10}}$

26. $16^{\frac{2}{3}} \cdot 16^{-\frac{1}{3}} = 16^{1/3}$

$= \sqrt[3]{16}$

$= \sqrt[3]{8} \cdot \sqrt[3]{2} = \boxed{2\sqrt[3]{2}}$

27. $(-216)^{-\frac{1}{3}} = \dfrac{1}{\sqrt[3]{-216}}$

$= \boxed{-\dfrac{1}{6}}$

28. $\left(\dfrac{112}{7}\right)^{-\frac{1}{4}} = \left(\dfrac{7}{112}\right)^{1/4}$

$= \left(\dfrac{1}{16}\right)^{1/4} = \sqrt[4]{\dfrac{1}{16}} = \boxed{\dfrac{1}{2}}$

29. $\sqrt[3]{v} \cdot \sqrt{v} = v^{1/3} \cdot v^{1/2}$

$= v^{5/6}$

$\boxed{= \sqrt[6]{v^5}}$

30. $\sqrt[4]{r^3} \cdot \sqrt{r} = r^{3/4} \cdot r^{1/2}$

$= r^{5/4}$

$= \sqrt[4]{r^5} = \sqrt[4]{r^4} \cdot \sqrt[4]{r}$

$\boxed{= r\sqrt[4]{r}}$

31. $\dfrac{4}{\sqrt[3]{4}} = \dfrac{4}{4^{1/3}} = 4^{2/3}$

$= \sqrt[3]{16}$

$= \sqrt[3]{8} \cdot \sqrt[3]{2} = \boxed{2\sqrt[3]{2}}$

32. $\dfrac{\sqrt{7^3}}{\sqrt{7}} = \dfrac{7^{3/2}}{7^{1/2}}$

$= 7^{2/2} = \boxed{7}$

33. $\sqrt[4]{x^{10}} = x^{10/4}$

$= x^{5/2}$

$= \sqrt{x^5}$

$= \sqrt{x^4}\sqrt{x} = \boxed{x^2\sqrt{x}}$

34. $\sqrt[4]{25m^2} = 25^{1/4} \cdot m^{1/2}$

$= (5^2)^{1/4} \cdot m^{1/2}$

$= 5^{1/2} \cdot m^{1/2}$

$= \boxed{\sqrt{5m}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

ADDING & SUBTRACTING *Radicals*

1. **SIMPLY** all radicals.
2. Identify radicals with the **SAME INDEX** and **SAME RADICAND.** Only these can be combined!
3. For common radicals, **add/subtract the coefficients** and **KEEP THE COMMON RADICAL.**

1. $3\sqrt{27}-2\sqrt{12}$

$3\sqrt{9}\sqrt{3} - 2\sqrt{4}\sqrt{3}$

$3\cdot 3\sqrt{3} - 2\cdot 2\sqrt{3}$

$9\sqrt{3} - 4\sqrt{3} = \boxed{5\sqrt{3}}$

2. $3\sqrt[3]{54}-2\sqrt[3]{2}+7\sqrt[3]{-16}$

$3\sqrt[3]{27}\sqrt[3]{2}$ $\quad 7\sqrt[3]{-8}\sqrt[3]{2}$

$3\cdot 3\sqrt[3]{2}$ $\quad 7\cdot(-2)\sqrt[3]{2}$

$9\sqrt[3]{2} - 2\sqrt[3]{2} - 14\sqrt[3]{2} = \boxed{-7\sqrt[3]{2}}$

3. $7\sqrt[4]{48}-2\sqrt[4]{3}+3\sqrt[3]{72}$

$7\sqrt[4]{16}\sqrt[4]{3}$ $\quad 3\sqrt[3]{8}\sqrt[3]{9}$

$7\cdot 2\sqrt[4]{3}$ $\quad 3\cdot 2\sqrt[3]{9}$

$14\sqrt[4]{3} - 2\sqrt[4]{3} + 6\sqrt[3]{9}$

$\boxed{= 12\sqrt[4]{3} + 6\sqrt[3]{9}}$

4. $10\sqrt{28}+\sqrt[3]{-56}-4\sqrt{175}$

$10\sqrt{4}\sqrt{7}$ $\quad \sqrt[3]{-8}\sqrt[3]{7}$ $\quad -4\sqrt{25}\sqrt{7}$

$10\cdot 2\sqrt{7}$ $\quad -2\sqrt[3]{7}$ $\quad -4\cdot 5\sqrt{7}$

$20\sqrt{7}$ $\quad -20\sqrt{7}$

$20\sqrt{7} - 2\sqrt[3]{7} - 20\sqrt{7} = \boxed{-2\sqrt[3]{7}}$

5. $\sqrt{98x^4y^2}-3x^2y\sqrt{2}$

$\sqrt{49x^4y^2}\sqrt{2}$

$7x^2y\sqrt{2} - 3x^2y\sqrt{2}$

$= \boxed{4x^2y\sqrt{2}}$

6. $\sqrt[3]{-40a^7}+2a^2\cdot\sqrt[3]{135a^4}$

$\sqrt[3]{-8a^6}\sqrt[3]{5a}$ $\quad 2a^2\sqrt[3]{27a^3}\sqrt[3]{5a}$

$-2a^2\sqrt[3]{5a}$ $\quad 2a^2\cdot 3a\sqrt[3]{5a}$

$= \boxed{-2a^2\sqrt[3]{5a} + 6a^3\sqrt[3]{5a}}$

MULTIPLYING *Radicals*

1. Multiply coefficients, then use the **PRODUCT RULE:** $\sqrt[n]{a}\cdot\sqrt[n]{b} = \sqrt[n]{a\cdot b}$
2. **SIMPLIFY** the resulting radical.

7. $\sqrt{27}\cdot\sqrt{5}$ $= \sqrt{135}$

$= \sqrt{9}\sqrt{15}$

$\boxed{= 3\sqrt{15}}$

8. $3\sqrt{10}\cdot -2\sqrt{18}$ $= -6\sqrt{180}$

$= -6\sqrt{36}\sqrt{5}$

$= -6\cdot 6\sqrt{5}$

$= \boxed{-36\sqrt{5}}$

9. $2\sqrt[3]{9}\cdot 5\sqrt[3]{-24}$ $= 10\sqrt[3]{-216}$

$= 10\cdot -6$

$= \boxed{-60}$

10. $-3\sqrt[4]{64}\cdot -\sqrt[4]{8}$ $= 3\sqrt[4]{512}$

$= 3\sqrt[4]{256}\sqrt[4]{2}$

$= 3\cdot 4\sqrt[4]{2} = \boxed{12\sqrt[4]{2}}$

11. $\sqrt{6x^4}\cdot 5\sqrt{8x^5}$

$5\sqrt{48x^9} = 5\sqrt{16x^8}\sqrt{3x}$

$= 5\cdot 4x^4\sqrt{3x}$

$= \boxed{20x^4\sqrt{3x}}$

12. $\sqrt[3]{54m^8}\cdot\sqrt[3]{5m^4}$

$\sqrt[3]{270m^{12}} = \sqrt[3]{27m^{12}}\sqrt[3]{10}$

$\boxed{= 3m^4\sqrt[3]{10}}$

13. $\sqrt[3]{-3a^7b^4}\cdot\sqrt[3]{36a^6b^2}$

$\sqrt[3]{-108a^{13}b^6}$

$= \sqrt[3]{-27a^{12}b^6}\sqrt[3]{4a}$

$= \boxed{-3a^4b^2\sqrt[3]{4a}}$

14. $2\sqrt[4]{p^2q}\cdot 7\sqrt[4]{p^3q^{10}}$

$14\sqrt[4]{p^5q^{11}}$

$= 14\sqrt[4]{p^4q^8}\sqrt[4]{pq^3}$

$\boxed{= 14pq^2\sqrt[4]{pq^3}}$

BINOMIAL
Examples

15. $\sqrt{10}(5\sqrt{5}-2\sqrt{2})$

$5\sqrt{50} - 2\sqrt{20}$

$= 5\sqrt{25}\sqrt{2} - 2\sqrt{4}\sqrt{5}$

$= 5\cdot 5\sqrt{2} - 2\cdot 2\sqrt{5}$

$= \boxed{25\sqrt{2} - 4\sqrt{5}}$

16. $(8-\sqrt{10})(3-\sqrt{10})$

$24 - 8\sqrt{10} - 3\sqrt{10} + \sqrt{100}$

$= 24 - 11\sqrt{10} + 10$

$= \boxed{34 - 11\sqrt{10}}$

17. $(6\sqrt{6}-6\sqrt{2})(\sqrt{6}+\sqrt{2})$

$6\sqrt{36} + 6\sqrt{12} - 6\sqrt{12} - 6\sqrt{4}$

$= 6\cdot 6 - 6\cdot 2$

$= 36 - 12$

$\boxed{= 24}$

18. $(4-\sqrt{5})^2$

$(4-\sqrt{5})(4-\sqrt{5})$

$= 16 - 4\sqrt{5} - 4\sqrt{5} + \sqrt{25}$

$= 16 - 8\sqrt{5} + 5$

$\boxed{= 21 - 8\sqrt{5}}$

19. $\sqrt{3k}\left(\sqrt{12k} - 2\sqrt{15k^2}\right)$

$\sqrt{36k^2} - 2\sqrt{45k^3}$

$= 6k - 2\sqrt{9k^2}\sqrt{5k}$

$= 6k - 2\cdot 3k\sqrt{5k}$

$= \boxed{6k - 6k\sqrt{5k}}$

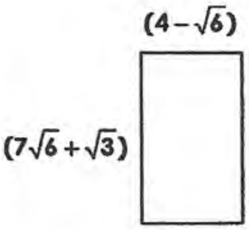
20. $(\sqrt{x}-\sqrt{8})(\sqrt{x}+\sqrt{2})$

$\sqrt{x^2} + \sqrt{2x} - \sqrt{8x} - \sqrt{16}$

$= x + \sqrt{2x} - \sqrt{4}\sqrt{2x} - 4$

$= x + \sqrt{2x} - 2\sqrt{2x} - 4$

$= \boxed{x - \sqrt{2x} - 4}$



21. Find the area and perimeter of the rectangle shown to the left.

Area: $(4-\sqrt{6})(7\sqrt{6}+\sqrt{3})$

$= 28\sqrt{6} + 4\sqrt{3} - 7\sqrt{36} - \sqrt{18}$

$= 28\sqrt{6} + 4\sqrt{3} - 7\cdot 6 - \sqrt{9}\sqrt{2}$

$\boxed{= 28\sqrt{6} + 4\sqrt{3} - 42 - 3\sqrt{2}}$

Perimeter:

$2(4-\sqrt{6}) + 2(7\sqrt{6}+\sqrt{3})$

$= 8 - 2\sqrt{6} + 14\sqrt{6} + 2\sqrt{3}$

$\boxed{= 8 + 12\sqrt{6} + 2\sqrt{3}}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
DIVIDING *Radicals*	① Divide coefficients, then use the **QUOTIENT RULE:** $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
	② **SIMPLIFY** the resulting radical.

1. $\frac{12\sqrt{160}}{2\sqrt{5}} = 6\sqrt{32} = 6\sqrt{16}\sqrt{2} = 6\cdot 4\sqrt{2} = \boxed{24\sqrt{2}}$

2. $\frac{36\sqrt[4]{1{,}250}}{9\sqrt[4]{2}} = 4\sqrt[4]{625} = 4\cdot 5 = \boxed{20}$

3. $\frac{\sqrt{x^3y^9}}{\sqrt{x^2y^5}} = \sqrt{xy^4} = \sqrt{y^4}\sqrt{x} = \boxed{y^2\sqrt{x}}$

4. $\frac{28\sqrt[3]{-16m^6}}{4\sqrt[3]{2m}} = 7\sqrt[3]{-8m^5} = 7\sqrt[3]{-8m^3}\sqrt[3]{m^2} = 7\cdot -2m\sqrt[3]{m^2} = \boxed{-14m\sqrt[3]{m^2}}$

5. $\sqrt{\frac{48}{16}} = \boxed{\sqrt{3}}$

6. $\sqrt[3]{\frac{40}{27}} = \frac{\sqrt[3]{8}\sqrt[3]{5}}{\sqrt[3]{27}} = \boxed{\frac{2\sqrt[3]{5}}{3}}$

7. $\sqrt[3]{\frac{7x^5}{64y^6}} = \frac{\sqrt[3]{x^3}\sqrt[3]{7x^2}}{\sqrt[3]{64y^6}} = \boxed{\frac{x\sqrt[3]{7x^2}}{4y^2}}$

8. $\sqrt[4]{\frac{32w}{81}} = \frac{\sqrt[4]{16}\sqrt[4]{2w}}{\sqrt[4]{81}} = \boxed{\frac{2\sqrt[4]{2w}}{3}}$

RATIONALIZING *the Denominator*

* **Monomial Demonimators:** Multiply the numerator and denominator by the radical.
* **Binomial Denominators:** Multiply the numerator and denominator by the conjugate. (The same expression but with the opposite sign in the middle.)

9. $\frac{3}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{2\cdot 5} = \boxed{\frac{3\sqrt{5}}{10}}$

10. $\sqrt{\frac{8}{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{120}}{\sqrt{225}} = \frac{\sqrt{4}\sqrt{30}}{15} = \boxed{\frac{2\sqrt{30}}{15}}$

	11. $\frac{3\sqrt{12}\cdot\sqrt{7}}{4\sqrt{7}\cdot\sqrt{7}} = \frac{3\sqrt{84}}{4\sqrt{49}}$ $= \frac{3\sqrt{4}\sqrt{21}}{4\cdot 7}$ $= \frac{6\sqrt{21}}{28} = \boxed{\frac{3\sqrt{21}}{14}}$	12. $\frac{\sqrt{32a^5}}{\sqrt{3a}} = \frac{\sqrt{32a^4}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$ $= \frac{\sqrt{96a^4}}{\sqrt{9}}$ $= \frac{\sqrt{16a^4}\sqrt{6}}{3}$ $= \boxed{\frac{4a^2\sqrt{6}}{3}}$
	13. $\frac{(\sqrt{8}-\sqrt{2})\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$ $= \frac{\sqrt{16}-\sqrt{4}}{\sqrt{4}}$ $= \frac{4-2}{2} = \boxed{1}$	14. $\frac{(3\sqrt{2}+\sqrt{6})\cdot\sqrt{12}}{\sqrt{12}\cdot\sqrt{12}}$ $= \frac{3\sqrt{24}+\sqrt{72}}{\sqrt{144}}$ $= \frac{3\sqrt{4}\sqrt{6}+\sqrt{36}\sqrt{2}}{12}$ $= \frac{6\sqrt{6}+6\sqrt{2}}{12} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{2}}$
BINOMIAL *Denominators*	15. $\frac{(4)(4-\sqrt{2})}{(4+\sqrt{2})(4-\sqrt{2})}$ $= \frac{16-4\sqrt{2}}{16-4\sqrt{2}+4\sqrt{2}-\sqrt{4}}$ $= \frac{16-4\sqrt{2}}{14} = \boxed{\frac{8-2\sqrt{2}}{7}}$	16. $\frac{(2)(6+\sqrt{5})}{(6-\sqrt{5})(6+\sqrt{5})}$ $= \frac{12+2\sqrt{5}}{36+6\sqrt{5}-6\sqrt{5}-\sqrt{25}}$ $= \boxed{\frac{12+2\sqrt{5}}{31}}$
	17. $\frac{(\sqrt{3})(1+4\sqrt{2})}{(1-4\sqrt{2})(1+4\sqrt{2})}$ $= \frac{\sqrt{3}+4\sqrt{6}}{1+4\sqrt{2}-4\sqrt{2}-16\sqrt{4}}$ $= \frac{\sqrt{3}+4\sqrt{6}}{-31} = \boxed{\frac{-\sqrt{3}-4\sqrt{6}}{31}}$	18. $\frac{(5-\sqrt{5})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$ $= \frac{5\sqrt{5}-5\sqrt{3}-\sqrt{25}+\sqrt{15}}{\sqrt{25}-\sqrt{15}+\sqrt{15}-\sqrt{9}}$ $= \boxed{\frac{5\sqrt{5}-5\sqrt{3}-5+\sqrt{15}}{2}}$
	19. $\frac{(3+\sqrt{7})(2-2\sqrt{7})}{(2+2\sqrt{7})(2-2\sqrt{7})}$ $= \frac{6-6\sqrt{7}+2\sqrt{7}-2\sqrt{49}}{4-4\sqrt{7}+4\sqrt{7}-4\sqrt{49}}$ $= \frac{6-4\sqrt{7}-14}{4-28} = \frac{-8-4\sqrt{7}}{-24}$ $= \boxed{\frac{2+\sqrt{7}}{6}}$	20. $\frac{(8-\sqrt{6})(3+4\sqrt{6})}{(3-4\sqrt{6})(3+4\sqrt{6})}$ $= \frac{24+32\sqrt{6}-3\sqrt{6}-4\sqrt{36}}{9+12\sqrt{6}-12\sqrt{6}-16\sqrt{36}}$ $= \frac{24+29\sqrt{6}-24}{9-96}$ $= \frac{29\sqrt{6}}{-87} = \boxed{-\frac{\sqrt{6}}{3}}$

REVIEW: Radicals & Rational Exponents

SIMPLIFYING RADICALS

Write each expression in simplest radical form.

1. $\sqrt{300m^4n^{25}}$	2. $\sqrt[3]{-72a^9b^2}$	3. $\sqrt[4]{81m^{11}n^{20}}$
$\sqrt{100m^4n^{24}}\ \sqrt{3n}$	$\sqrt[3]{-8a^9}\ \sqrt[3]{9b^2}$	$\sqrt[4]{81m^8n^{20}}\ \sqrt[4]{m^3}$
$=\boxed{10m^2n^{12}\sqrt{3n}}$	$=\boxed{-2a^3\sqrt[3]{9b^2}}$	$=\boxed{3m^2n^5\sqrt[4]{m^3}}$

OPERATIONS WITH RADICALS

Perform the indicated operation(s). Write each answer in simplest radical form.

4. $4\sqrt{98}+\sqrt{150}-2\sqrt{32}$

$4\sqrt{49}\sqrt{2}+\sqrt{25}\sqrt{6}-2\sqrt{16}\sqrt{2}$

$=28\sqrt{2}+5\sqrt{6}-8\sqrt{2}$

$=\boxed{20\sqrt{2}+5\sqrt{6}}$

5. $10\sqrt[4]{5}+7\sqrt[3]{40}-\sqrt[3]{320}$

$10\sqrt[4]{5}+7\sqrt[3]{8}\sqrt[3]{5}-\sqrt[3]{64}\sqrt[3]{5}$

$=10\sqrt[4]{5}+14\sqrt[3]{5}-4\sqrt[3]{5}$

$=\boxed{10\sqrt[4]{5}+10\sqrt[3]{5}}$

6. $3\sqrt[4]{8}\cdot 7\sqrt[4]{32}$

$21\sqrt[4]{256}$

$=21\cdot 4\quad =\boxed{84}$

7. $\sqrt[3]{-6x^5}\cdot\sqrt[3]{18x^4}$

$\sqrt[3]{-108x^9}\quad=\sqrt[3]{-27x^9}\ \sqrt[3]{4}$

$=\boxed{-3x^3\sqrt[3]{4}}$

8. $\sqrt{3}(2-\sqrt{24})$

$2\sqrt{3}-\sqrt{72}$

$=2\sqrt{3}-\sqrt{36}\sqrt{2}$

$=\boxed{2\sqrt{3}-6\sqrt{2}}$

9. $(4+3\sqrt{5})\cdot(-2+\sqrt{5})$

$-8+4\sqrt{5}-6\sqrt{5}+3\sqrt{25}$

$=-8-2\sqrt{5}+15$

$=\boxed{7-2\sqrt{5}}$

10. $(1+7\sqrt{3})(1-7\sqrt{3})$

$1-7\sqrt{3}+7\sqrt{3}-49\sqrt{9}$

$1-147$

$=\boxed{-146}$

11. $(3+\sqrt{6})^2$

$(3+\sqrt{6})(3+\sqrt{6})$

$=9+3\sqrt{6}+3\sqrt{6}+\sqrt{36}$

$=9+6\sqrt{6}+6\quad=\boxed{15+6\sqrt{6}}$

12. $\dfrac{-45\sqrt{156}}{9\sqrt{3}}$

$=-5\sqrt{52}$

$=-5\sqrt{4}\sqrt{13}$

$=\boxed{-10\sqrt{13}}$

13. $\dfrac{\sqrt[4]{r^{20}s^9}}{\sqrt[4]{r^3s}}=\sqrt[4]{r^{17}s^8}$

$=\sqrt[4]{r^{16}s^8}\ \sqrt[4]{r}$

$=\boxed{r^4s^2\sqrt[4]{r}}$

14. $\sqrt{\frac{50}{81}} = \frac{\sqrt{25}\,\sqrt{2}}{\sqrt{81}} = \boxed{\frac{5\sqrt{2}}{9}}$

15. $\sqrt[4]{\frac{3a}{16b^8}}$ $\quad \frac{\sqrt[4]{3a}}{\sqrt[4]{16b^8}} = \boxed{\frac{\sqrt[4]{3a}}{2b^2}}$

16. $\frac{\sqrt{6}}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{18}}{4\sqrt{9}}$

$= \frac{\sqrt{9}\,\sqrt{2}}{4\sqrt{9}} = \boxed{\frac{\sqrt{2}}{4}}$

17. $\sqrt{\frac{8x}{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{40x}}{\sqrt{25}} = \frac{\sqrt{4}\,\sqrt{10x}}{5}$

$= \boxed{\frac{2\sqrt{10x}}{5}}$

18. $\frac{(2-\sqrt{12})\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{36}}{\sqrt{9}}$

$= \boxed{\frac{2\sqrt{3}-6}{3}}$

19. $\frac{3+\sqrt{3}}{4\sqrt{12}} = \frac{3+\sqrt{3}}{4\sqrt{4}\sqrt{3}} = \frac{(3+\sqrt{3})\cdot\sqrt{3}}{(8\sqrt{3})\cdot\sqrt{3}}$

$= \frac{3\sqrt{3}+\sqrt{9}}{8\sqrt{9}} = \boxed{\frac{\sqrt{3}+1}{8}}$

20. $\frac{(5-4\sqrt{2})(4+3\sqrt{2})}{(4-3\sqrt{2})(4+3\sqrt{2})}$

$= \frac{20+15\sqrt{2}-16\sqrt{2}-12\sqrt{4}}{16+12\sqrt{2}-12\sqrt{2}-9\sqrt{4}}$

$= \frac{20-\sqrt{2}-24}{16-18} = \frac{-4-\sqrt{2}}{-2} = \boxed{\frac{4+\sqrt{2}}{2}}$

21. $\frac{(2+3\sqrt{3})(\sqrt{3}-5)}{(\sqrt{3}+5)(\sqrt{3}-5)}$

$= \frac{2\sqrt{3}-10+3\sqrt{9}-15\sqrt{3}}{\sqrt{9}-5\sqrt{3}+5\sqrt{3}-25}$

$= \frac{-13\sqrt{3}-10+9}{-22} = \frac{-1-13\sqrt{3}}{-22} = \boxed{\frac{1+13\sqrt{3}}{22}}$

RATIONAL EXPONENTS

Simplify each expression. Write all final answers in simplest radical form.

22. $x^{\frac{2}{3}} \cdot x^{\frac{11}{3}} = x^{13/3}$

$= \sqrt[3]{x^{13}}$

$= \sqrt[3]{x^{12}} \cdot \sqrt[3]{x} = \boxed{x^4\sqrt[3]{x}}$

23. $32^{\frac{1}{8}} \cdot 32^{\frac{3}{8}} = 32^{4/8}$

$= 32^{1/2}$

$= \sqrt{32}$

$= \sqrt{16}\,\sqrt{2} = \boxed{4\sqrt{2}}$

24. $n^{\frac{7}{3}} \cdot n^3 = n^{\frac{16}{3}}$

$= \sqrt[3]{n^{16}}$

$= \sqrt[3]{n^{15}}\,\sqrt[3]{n} = \boxed{n^5\sqrt[3]{n}}$

25. $\frac{7^{\frac{9}{4}}}{7^{\frac{1}{4}}} = 7^{8/4} = 7^2$

$\boxed{=49}$

26. $\left(k^{\frac{3}{2}}\right)^{\frac{1}{2}} = k^{3/4}$

$= \boxed{\sqrt[4]{k^3}}$

27. $1{,}296^{-\frac{1}{4}}$

$= \frac{1}{\sqrt[4]{1296}} = \boxed{\frac{1}{6}}$

28. $\frac{125^{\frac{1}{3}}}{125^{\frac{2}{3}}} = \frac{1}{125^{1/3}}$

$= \frac{1}{\sqrt[3]{125}} = \boxed{\frac{1}{5}}$

29. $\frac{\sqrt[4]{3^3}}{\sqrt{3}} = \frac{3^{3/4}}{3^{1/2}} = 3^{1/4}$

$= \boxed{\sqrt[4]{3}}$

30. $\sqrt[4]{10^2a^{18}}$

$(10^2a^{18})^{1/4}$

$= 10^{1/2}a^{9/2}$

$= \boxed{a^4\sqrt{10a}}$

Kuta Software - Infinite Algebra 2 Name________________________________

Factoring: All Techniques Combined [redacted]

Date________________ Period____

Factor each. Week 8 - [redacted] (3 problems/day)

1) $x^3 - 5x^2 - x + 5$

2) $x^4 - 2x^2 - 15$

3) $x^6 - 26x^3 - 27$

4) $x^6 + 2x^4 - 16x^2 - 32$

5) $x^4 - 13x^2 + 40$

6) $x^9 - x^6 - x^3 + 1$

7) $x^6 - 4x^2$

8) $x^4 + 14x^2 + 45$

9) $2x^4 + x^2 - 6$

10) $2x^2 - 13x + 20$

11) $4x^3 - x^2 - 4x + 1$

12) $4x^8 - 61x^4 + 225$

13) $5x^2 + 24x - 5$

14) $5x^2 + 29x + 20$

15) $4x^2 + 4x - 15$

16) $10x^3 - 8x^2 + 25x - 20$

17) $-64x^3 + 125 = 0$

18) $8x^4 + 10x^2 - 3$

49. $4n^3-20n^2+25n$

$n(4n^2-20n+25)$

$\boxed{n(2n-5)(2n-5)}$

50. $4c^2d^2-16d^2$

$4d^2(c^2-4)$

$\boxed{4d^2(c+2)(c-2)}$

51. $2k^2-8k-90$

$2(k^2-4k-45)$

$\boxed{2(k-9)(k+5)}$

52. y^5-9y^4-y+9

$y^4(y-9)-1(y-9)$

$(y^4-1)(y-9)$

$(y^2-1)(y^2+1)(y-9)$

$\boxed{(y+1)(y-1)(y^2+1)(y-9)}$

GCF

(Greatest Common Factor)

EXAMPLES

1. $8k-32$ — $\boxed{8(k-4)}$

2. $21w-6$ — $\boxed{3(7w-2)}$

3. $10p^2-2p$ — $\boxed{2p(5p-1)}$

4. $m^5n-3m^2n^4$ — $\boxed{m^2n(m^3-3n^3)}$

5. $18a^2b+27abc$ — $\boxed{9ab(2a+3c)}$

6. $8x^3-36x^2y-20xy$ — $\boxed{4x(2x^2-9xy-5y)}$

B

DIFFERENCE OF SQUARES

a^2-b^2

TO FACTOR: $(a+b)(a-b)$

EXAMPLES

7. x^2-4 — $\boxed{(x+2)(x-2)}$

8. m^2-81 — $\boxed{(m+9)(m-9)}$

9. $16-p^2$ — $\boxed{(4+p)(4-p)}$

10. $49a^2-1$ — $\boxed{(7a+1)(7a-1)}$

11. w^6-25v^2 — $\boxed{(w^3+5v)(w^3-5v)}$

12. $100r^4-9s^4$ — $\boxed{(10r^2-3s^2)(10r^2+3s^2)}$

45. k^3+k^2-6k-6

$k^2(k+1)-6(k+1)$

$\boxed{(k^2-6)(k+1)}$

46. $10w^2-w-3$

w^2-w-30

$(w-\frac{6}{10})(w+\frac{5}{10})$

$(w-\frac{3}{5})(w+\frac{1}{2})$

$\boxed{(5w-3)(2w+1)}$

47. x^5y-xy^5

$xy(x^4-y^4)$

$xy(x^2+y^2)(x^2-y^2)$

$\boxed{xy(x+y)(x-y)(x^2+y^2)}$

48. $3r^3+4r^2-12r-16$

$r^2(3r+4)-4(3r+4)$

$(r^2-4)(3r+4)$

$\boxed{(r+2)(r-2)(3r+4)}$

TRINOMIALS

use "X's method"

$ax^2 + bx + c \quad (a = 1)$

TO FACTOR:
Find factors that multiply to "c" & add to "b"

EXAMPLES

18. $x^2 + 6x + 8$
$\boxed{(x+2)(x+4)}$

19. $m^2 + 13m + 40$
$\boxed{(m+8)(m+5)}$

20. $p^2 - 19p + 84$
$\boxed{(p-7)(p-12)}$

21. $c^2 - 2c + 1$
$\boxed{(c-1)(c-1)}$

22. $v^2 + 2v - 15$
$\boxed{(v+5)(v-3)}$

23. $k^2 + 9k - 90$
$\boxed{(k+15)(k-6)}$

38. $m^3 - 3m^2n - 8mn + 24n^2$
$m^2(m-3n) - 8n(m-3n)$
$\boxed{(m^2-8n)(m-3n)}$

EXAMPLES WITH DOS

39. $p^3 - 4p^2 - 9p + 36$
$p^2(p-4) - 9(p-4)$
$(p^2-9)(p-4)$
$\boxed{(p-3)(p+3)(p-4)}$

40. $8w^3 + 12w^2 - 2w - 3$
$4w^2(2w+3) - 1(2w+3)$
$(4w^2-1)(2w+3)$
$\boxed{(2w-1)(2w+1)(2w+3)}$

MIXED PRACTICE

STEP 1: FACTOR A GCF (IF THERE IS ONE)

STEP 2: CHECK NUMBER OF TERMS & FACTOR
- Binomial? (try DOS!)
- Trinomial?
- Four Terms?

STEP 3: CAN YOU KEEP FACTORING?

WHAT IF IT CAN'T BE FACTORED AT ALL?
It's prime!

41. $10x - 4y^2$
$\boxed{2(5x-2y^2)}$

42. $7a^2 - 28b^2$
$7(a^2-4b^2)$
$\boxed{7(a+2b)(a-2b)}$

43. $2v^2 - 22v + 36$
$2(v^2-11v+18)$
$\boxed{2(v-9)(v-2)}$

44. $m^2 - 8n^2$
$\boxed{\text{Prime}}$

EXAMPLES WITH GCF

13. $5n^2 - 5$
$5(n^2-1)$
$\boxed{5(n+1)(n-1)}$

14. $4k^2 - 36$
$4(k^2-9)$
$\boxed{4(k+3)(k-3)}$

15. $12w^5 - 75w^3$
$3w^3(4w^2-25)$
$\boxed{3w^3(2w+5)(2w-5)}$

16. $36pq - p^5q$
$pq(36-p^4)$
$\boxed{pq(6-p^2)(6+p^2)}$

17. Give three examples of binomials that are NOT a difference of squares.
$x^2 - 15$
$x^4 + 9$
$x^3 - 9$

TRINOMIALS

$ax^2 + bx + c \quad (a > 1)$

Use X method

TO FACTOR: ~~slide + divide~~

EXAMPLES

30 $3x^2 - 2x - 8$

$x^2 - 2x - 24$

$(x - \frac{6}{3})(x + \frac{4}{3})$

$\boxed{(x-2)(3x+4)}$

31 $4n^2 - 12n + 5$

$n^2 - 12n + 20$

$(n - \frac{2}{4})(n - \frac{10}{4})$

$(n - \frac{1}{2})(n - \frac{5}{2})$

$\boxed{(2n-1)(2n-5)}$

32 $6a^2 - 5a - 21$

$a^2 - 5a - 126$

$(a - \frac{14}{6})(a + \frac{9}{6})$

$(a - \frac{7}{3})(a + \frac{3}{2})$ $\boxed{(3a-7)(2a+3)}$

33 $16w^2 - 8w + 1$

$w^2 - 8w + 16$

$(w - \frac{4}{16})(w - \frac{4}{16})$

$(w - \frac{1}{4})(w - \frac{1}{4})$

$\boxed{(4w-1)(4w-1)}$

EXAMPLE WITH GCF

34 $15y^2 + 21y + 6$

$3(5y^2 + 7y + 2)$

$3(y^2 + 7y + 10)$

$3(y + \frac{5}{5})(y + \frac{2}{5})$

$\boxed{3(y+1)(5y+2)}$

FOUR TERMS

TO FACTOR: split into 2 binomials, find GCF of each

EXAMPLES

35 $x^3 - 4x^2 + 5x - 20$

$x^2(x-4) + 5(x-4)$

$\boxed{(x^2+5)(x-4)}$

36 $6y^3 + 27y^2 - 2y - 9$

$3y^2(2y+9) - 1(2y+9)$

$\boxed{(3y^2-1)(2y+9)}$

37 $4a^3 - 24a^2 - 7a + 42$

$4a^2(a-6) - 7(a-6)$

$\boxed{(4a^2-7)(a-6)}$

24 $r^2 - r - 30$

$\boxed{(r-6)(r+5)}$

25 $b^2 - 11b - 42$

$\boxed{(b-14)(b+3)}$

EXAMPLES WITH GCF

26 $5n^2 + 40n + 80$

$5(n^2 + 8n + 16)$

$\boxed{5(n+4)(n+4)}$

27 $2k^2 - 6k - 56$

$2(k^2 - 3k - 28)$

$\boxed{2(k-7)(k+4)}$

28 $x^3 - 12x^2 + 27x$

$x(x^2 - 12x + 27)$

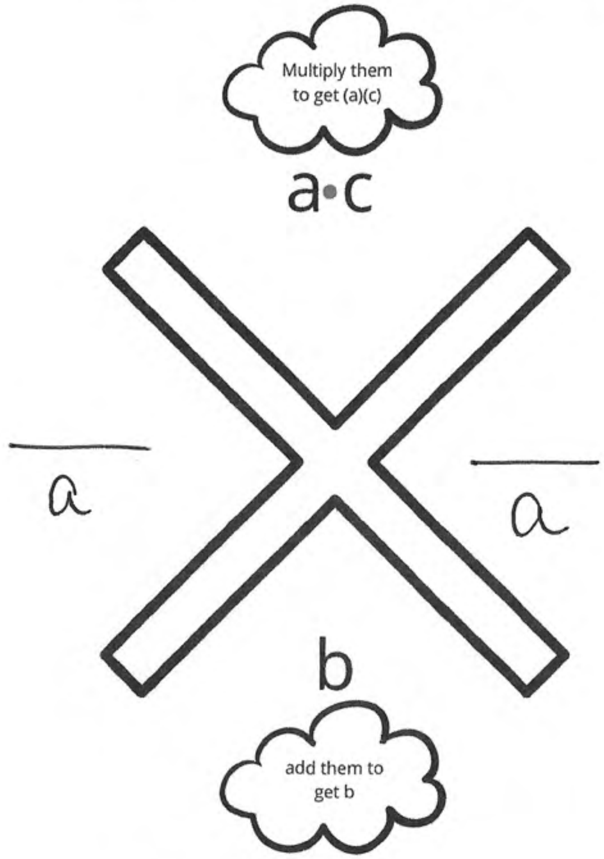
$\boxed{x(x-9)(x-3)}$

29 $3a^2b + 15ab - 42b$

$3b(a^2 + 5a - 14)$

$\boxed{3b(a+7)(a-2)}$

How to use the x method for factoring – for trinomials



write all the factors for a•c here

$x^2 + 3x + 2$

$ax^2 + bx + c$

a=

b=

c=

= (   )(   )